

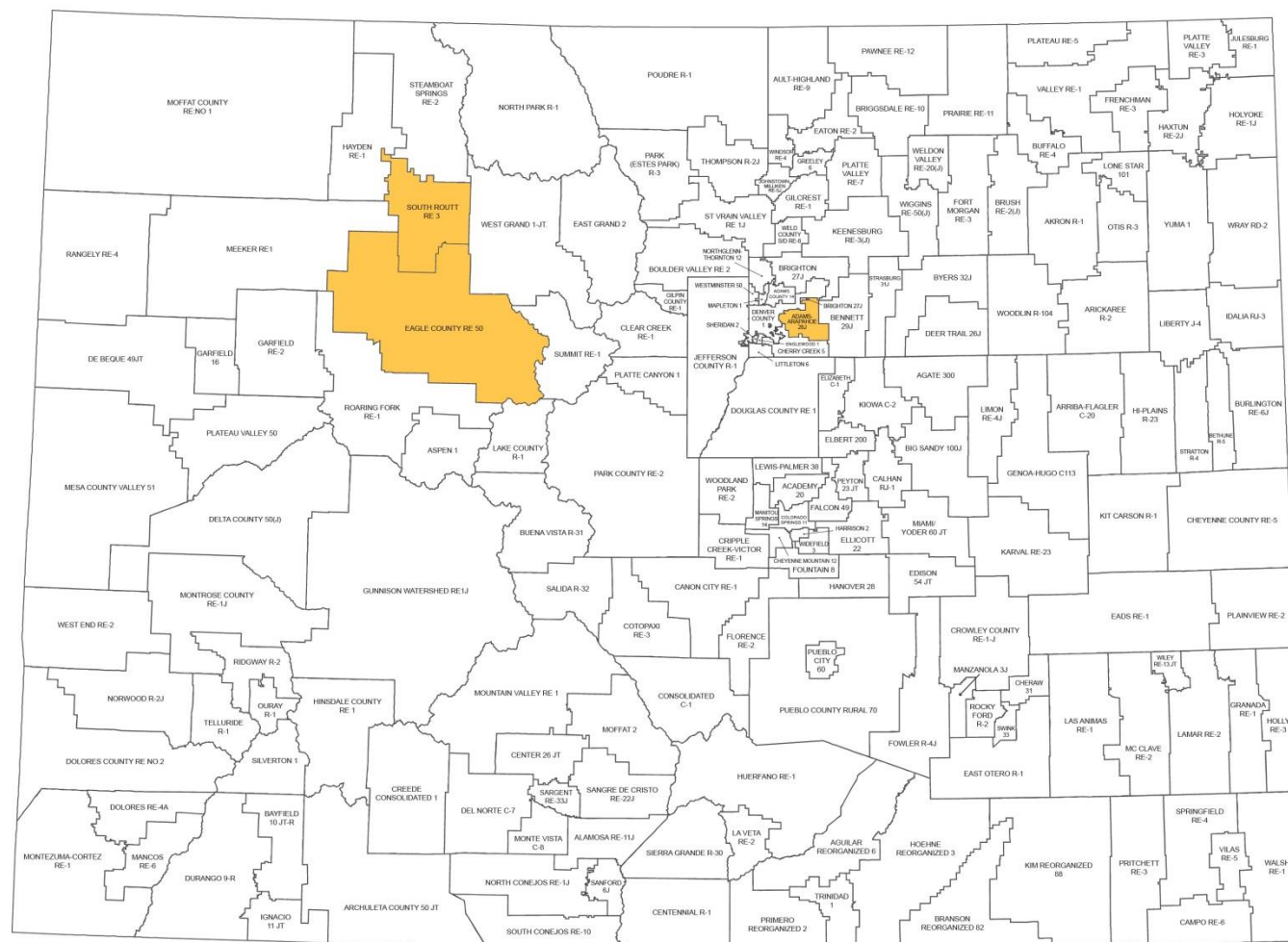
# Unit Title: Transform the World

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## BASED ON A CURRICULUM OVERVIEW SAMPLE AUTHORED BY

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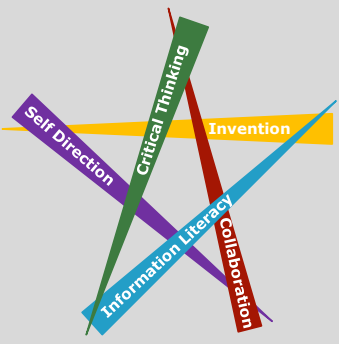


*This unit was authored by a team of Colorado educators. The template provided one example of unit design that enabled teacher-authors to organize possible learning experiences, resources, differentiation, and assessments. The unit is intended to support teachers, schools, and districts as they make their own local decisions around the best instructional plans and practices for all students.*

**Colorado Teacher-Authored Sample Instructional Unit**

<b>Content Area</b>	Mathematics	<b>Grade Level</b>	High School
<b>Course Name/Course Code</b>	Mathematics 1		
<b>Standard</b>	<b>Grade Level Expectations (GLE)</b>	<b>GLE Code</b>	
1. Number Sense, Properties, and Operations	1. The complex number system includes real numbers and imaginary numbers	MA10-GR.HS-S.1-GLE.1	
	2. Quantitative reasoning is used to make sense of quantities and their relationships in problem situations	MA10-GR.HS-S.1-GLE.2	
2. Patterns, Functions, and Algebraic Structures	1. Functions model situations where one quantity determines another and can be represented algebraically, graphically, and using tables	MA10-GR.HS-S.2-GLE.1	
	2. Quantitative relationships in the real world can be modeled and solved using functions	MA10-GR.HS-S.2-GLE.2	
	3. Expressions can be represented in multiple, equivalent forms	MA10-GR.HS-S.2-GLE.3	
	4. Solutions to equations, inequalities and systems of equations are found using a variety of tools	MA10-GR.HS-S.2-GLE.4	
3. Data Analysis, Statistics, and Probability	1. Visual displays and summary statistics condense the information in data sets into usable knowledge	MA10-GR.HS-S.3-GLE.1	
	2. Statistical methods take variability into account supporting informed decisions making through quantitative studies designed to answer specific questions	MA10-GR.HS-S.3-GLE.2	
	3. Probability models outcomes for situations in which there is inherent randomness	MA10-GR.HS-S.3-GLE.3	
4. Shape, Dimension, and Geometric Relationships	1. Objects in the plane can be transformed, and those transformations can be described and analyzed mathematically	MA10-GR.HS-S.4-GLE.1	
	2. Concepts of similarity are foundational to geometry and its applications	MA10-GR.HS-S.4-GLE.2	
	3. Objects in the plane can be described and analyzed algebraically	MA10-GR.HS-S.4-GLE.3	
	4. Attributes of two- and three-dimensional objects are measurable and can be quantified	MA10-GR.HS-S.4-GLE.4	
	5. Objects in the real world can be modeled using geometric concepts	MA10-GR.HS-S.4-GLE.5	

**Colorado 21<sup>st</sup> Century Skills**



**Critical Thinking and Reasoning:** *Thinking Deeply, Thinking Differently*

**Information Literacy:** *Untangling the Web*

**Collaboration:** *Working Together, Learning Together*

**Self-Direction:** *Own Your Learning*

**Invention:** *Creating Solutions*

**Mathematical Practices:**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

<b>Unit Titles</b>	<b>Length of Unit/Contact Hours</b>	<b>Unit Number/Sequence</b>
Transform the World	8 weeks	6

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<b>Unit Title</b>	Transform the World	<b>Length of Unit</b>	8 weeks
<b>Focusing Lens(es)</b>	Justification Transformation	<b>Standards and Grade Level Expectations Addressed in this Unit</b>	MA10-GR.HS-S.4-GLE.1
<b>Inquiry Questions (Engaging-Debatable):</b>	<ul style="list-style-type: none"> <li>How do architectural engineers use transformations? (MA10-GR.HS-S.4-GLE.1)</li> </ul>		
<b>Unit Strands</b>	Geometry: Congruence		
<b>Concepts</b>	Algebraic representations, model, transformation, coordinate plane, angles, side lengths, congruency, definitions, proofs		

<b>Generalizations</b> My students will <b>Understand</b> that...	<b>Guiding Questions</b>	
	<b>Factual</b>	<b>Conceptual</b>
Algebraic representations model geometric transformations performed on a coordinate plane. (MA10-GR.HS-S.4-GLE.1-EO.a)	On a coordinate plane, what algebraic description describes a translation? Rotation? Reflection? What is the effect on the x and y coordinates of a point when applying rotations, reflections or translations?	Why is it useful to describe transformations on a coordinate plane? How is it possible for different compositions of transformations to be equivalent? Why is a rotation of 180 degrees equivalent to a reflection over the x-axis combined with a reflection over the y-axis?
Sequences of transformations or combinations of angles and side lengths can determine the congruence of shapes. (MA10-GR.HS-S.4-GLE.1-EO.b)	What transformations preserve shape and size? How can you determine if a transformation preserves shape and size? What combinations of sides and angles are sufficient to prove congruency of triangles? How can congruence between shapes be shown through indirect comparison?	Why can transformations determine if two figures are congruent? Why do combinations of sides and angles prove congruency of triangles? Why are some combinations of angles and sides sufficient to prove congruency while others are not?
Precise definitions of basic geometric concepts facilitate the development of careful proofs. (MA10-GR.HS-S.4-GLE.1-EO.c)	What is an angle? What is a triangle? What is a parallelogram?	How do definitions contribute to the development of a proof?

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<b>Key Knowledge and Skills:</b> <b>My students will...</b>	<i>What students will know and be able to do are so closely linked in the concept-based discipline of mathematics. Therefore, in the mathematics samples what students should know and do are combined.</i>
<ul style="list-style-type: none"> <li>• Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. (MA10-GR.HS-S.4-GLE.1-EO.a.i)</li> <li>• Represent transformations in the plane and describe transformations as functions that take points in the plane as inputs and give other points as outputs. (MA10-GR.HS-S.4-GLE.1-EO.a.ii, iii)</li> <li>• Compare transformations that preserve distance and angle to those that do not. (MA10-GR.HS-S.4-GLE.1-EO.a.iv)</li> <li>• Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. (MA10-GR.HS-S.4-GLE.1-EO.a.v)</li> <li>• Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. (MA10-GR.HS-S.4-GLE.1-EO.a.vi)</li> <li>• Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure. (MA10-GR.HS-S.4-GLE.1-EO.a.vii)</li> <li>• Specify a sequence of transformations that will carry a given figure onto another. (MA10-GR.HS-S.4-GLE.1-EO.a.viii)</li> <li>• Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. (MA10-GR.HS-S.4-GLE.1-EO.b.i, ii)</li> <li>• Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. ((MA10-GR.HS-S.4-GLE.1-EO.b.iii)</li> <li>• Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. (MA10-GR.HS-S.4-GLE.1-EO.b.iv)</li> <li>• Prove theorems about lines and angles, triangles, and parallelograms. (MA10-GR.HS-S.4-GLE.1-EO.c)</li> </ul>	

<p><b>Critical Language:</b> includes the Academic and Technical vocabulary, semantics, and discourse which are particular to and necessary for accessing a given discipline.          EXAMPLE: A student in Language Arts can demonstrate the ability to apply and comprehend critical language through the following statement: <i>“Mark Twain exposes the hypocrisy of slavery through the use of satire.”</i></p>	
<p><b>A student in _____ can demonstrate the ability to apply and comprehend critical language through the following statement(s):</b></p>	<p><i>I can use rigid transformations to show that necessary and sufficient combinations of congruent sides and angles prove triangles congruent.</i></p>
<p><b>Academic Vocabulary:</b></p>	<p>Classify, identify, compare, analyze, prove, substitution, develop, sufficient, necessary, definition, coordinate plane, angles, side lengths,</p>
<p><b>Technical Vocabulary:</b></p>	<p>Transformation, definitions, proofs, vertical angles, perpendicular bisector, rotation, translation, reflection, rigid transformation, congruence, theorem, postulate,</p>

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<b>Unit Description:</b>	This unit investigates transformations, congruence, and proof. Students begin with informal explorations of rigid transformations. Students then use precise mathematical definitions of rotations, translations and reflections to determine if each of these rigid transformations is a function. The transformation work of this unit leads students to develop formal proofs about congruency, parallel lines, triangle relationships, and parallelograms.
<b>Unit Generalizations</b>	
<b>Key Generalization:</b>	Precise definitions of basic geometric concepts facilitate the development of careful proofs
<b>Supporting Generalizations:</b>	Algebraic representations model geometric transformations performed on a coordinate plane Sequences of transformations or combinations of angles and side lengths can determine the congruence of shapes

<b>Performance Assessment: <i>The capstone/summative assessment for this unit.</i></b>	
<b>Claims:</b> (Key generalization(s) to be mastered and demonstrated through the capstone assessment.)	Precise definitions of basic geometric concepts facilitate the development of careful proofs.
<b>Stimulus Material:</b> (Engaging scenario that includes role, audience, goal/outcome and explicitly connects the key generalization)	You are an artist and the mathematics museum ( <a href="http://momath.org/">http://momath.org/</a> ) has created a contest for a geometric mural. They want the mural to allow visitors to create geometric proofs about polygons and congruence. Your mural will need to have triangles and other polygons that can be shown to be congruent through transformations and other geometric theorems. The mural design should include symbols for congruence, perpendicularity, and parallelism as appropriate. The mural exhibit will also need to include questions for visitors to answer and information placards with answers and proofs for each question. For example, why are the green and red triangles congruent?
<b>Product/Evidence:</b> (Expected product from students)	Students will create a mural, set of questions about the mural, and answers to each question. The design of the mural could include: <ul style="list-style-type: none"> <li>• At least two pairs of triangles that can be proved congruent through transformations, ASA or SAS.</li> <li>• A visual of at least one other theorem from the unit such as angles formed by parallel lines cut by a transversal.</li> </ul> The questions about the mural could include concepts such as: <ul style="list-style-type: none"> <li>• Transformations</li> <li>• Congruency</li> </ul> The answers to each question should include clear explanations, proofs and visuals to help visitors understand each mathematical concept.
<b>Differentiation:</b> (Multiple modes for student expression)	Students can write their questions and corresponding answers using their notes from this unit as scaffolding. Student can write their questions and corresponding answers using proofs not discussed in this unit as an extension. Students can use drafting tools including software ( <a href="http://www.techsupportalert.com/best-free-cad-program.htm">http://www.techsupportalert.com/best-free-cad-program.htm</a> ) to create their designs and make interactive murals using touch screens.

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<b>Texts for independent reading or for class read aloud to support the content</b>	
<b>Informational/Non-Fiction</b>	<b>Fiction</b>
<i>Simon &amp; Schuster's Guide to Gems and Precious Stones</i> by Simon & Schuster and Kennie Lyman (Lexile level 800+)	<i>The Flying Circus of Physics</i> by Jearl Walker (Lexile level 800+) <i>Flatland: A Romance of Many Dimensions</i> by Edwin Abbot (Lexile level 1280)

<b>Ongoing Discipline-Specific Learning Experiences</b>				
1.	Description:	Think/work like a mathematician – Expressing mathematical reasoning by constructing viable arguments, critiquing the reasoning of others	Teacher Resources:	<a href="http://www.insidemathematics.org/index.php/standard-3">http://www.insidemathematics.org/index.php/standard-3</a> (examples of constructing viable arguments) <a href="http://quizlet.com/22134361/cpm-index-cards-of-teaching-strategies-flash-cards/">http://quizlet.com/22134361/cpm-index-cards-of-teaching-strategies-flash-cards/</a> (teaching strategies to encourage class discussions)
			Student Resources:	N/A
	Skills:	Provide justification for arguments through a series of logical steps while using correct mathematical vocabulary. Analyze and critique the arguments of other students	Assessment:	Students provide valid geometric proofs by justifying their reasoning to peers on a consistent basis. Students can also critique a fellow student’s proof for validity. Students will need to be precise with language such as definitions, symbols, and algebraic descriptions to demonstrate a high quality proof.
2.	Description:	Think/work like a mathematician – Engaging in the practice of modeling the solution to real world problems	Teacher Resources:	<a href="http://www.corestandards.org/Math/Content/HSM">http://www.corestandards.org/Math/Content/HSM</a> (Common Core State Standards description of the modeling process) <a href="http://blog.mrmeyer.com/?p=16301">http://blog.mrmeyer.com/?p=16301</a> (Dan Meyer discussion on modeling) <a href="http://threeacts.mrmeyer.com">http://threeacts.mrmeyer.com</a> (Examples of 3-act problems)
			Student Resources:	<a href="http://learni.st/users/S33572/boards/2771-defining-transformations-using-basic-geometry-objects-common-core-standard-9-12-g-co-4">http://learni.st/users/S33572/boards/2771-defining-transformations-using-basic-geometry-objects-common-core-standard-9-12-g-co-4</a> (video 10 is titled “Applications of Transformations” and examines how transformations are used in animations)
	Skills:	Model real world problems mapping relationships with appropriate models, analyze relationships to draw conclusions, interpret results in relation to context, justify and defend the model, and reflect on whether results make sense	Assessment:	Modeling Problems Students use transformations to model real world contexts in the fields of geology, biology, architecture and animation. Students will be able to draw conclusions and interpret their models in relation to the context to determine if their model makes sense.

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3.	Description:	Mathematicians are fluent with geometric transformations	Teacher Resources:	N/A
			Student Resources:	<a href="http://www.mathplayground.com/ShapeMods/ShapeMods.html">http://www.mathplayground.com/ShapeMods/ShapeMods.html</a> (practice on a coordinate grid with reflections) <a href="http://www.kidsmathgamesonline.com/geometry/transformation.html">http://www.kidsmathgamesonline.com/geometry/transformation.html</a> (practice on a coordinate grid with reflections) <a href="http://learni.st/users/S33572/boards/2771-defining-transformations-using-basic-geometry-objects-common-core-standard-9-12-g-co-4">http://learni.st/users/S33572/boards/2771-defining-transformations-using-basic-geometry-objects-common-core-standard-9-12-g-co-4</a> (videos and practice problems with transformations)
	Skills:	Visualize geometric relationships between objects using transformations	Assessment:	Fluency Problems Students build fluency with geometric transformations with consistent practice visualizing the result of rotations, reflections and translations.

**Prior Knowledge and Experiences**

Student familiarity with informally rotating, translating, and reflecting objects will provide a strong foundation for this unit. It is also helpful for students to have a degree of comfort using a coordinate grid. Knowledge of the basic properties of polygons, parallel and perpendicular lines, angles, slope, midpoints of line segments, and the Pythagorean theorem will also support student learning, many of these ideas are formalized through proof in this unit.

**Learning Experience # 1**

The teacher may provide straight manipulatives (e.g., toothpicks, straws) so that students can begin to develop and deepen their usage of key terminology (e.g., parallel lines, perpendicular lines, line segments, triangle, parallelogram, angles, circle, circular arc, point, line).

*Enactive:* Students can represent each geometric concept with the manipulatives.

*Iconic:* Students can draw representations of each concept and non-examples of each geometric concept.

*Symbolic:* Students can create directions for drawing each geometric concept and then share the directions with another student to determine if the directions are clear and precise. Students can then use these directions as a starting point for determining clear and precise definitions for each geometric concept.

<b>Teacher Notes:</b>	This is a formative assessment of students' prior understanding of each of these concepts. Students should be encouraged to show each concept in multiple ways (e.g., triangles that are not equilateral) and also non-examples of each concept. Students should also practice verbalizing a description of each concept to their neighbor using precise language.
<b>Generalization Connection(s):</b>	Precise definitions of basic geometric concepts facilitate the development of careful proofs
<b>Teacher Resources:</b>	<a href="http://www.geometrycommoncore.com/content/unit1/gco1/webresources.html">http://www.geometrycommoncore.com/content/unit1/gco1/webresources.html</a> (resources for definitions associated with this standard)
<b>Student Resources:</b>	<a href="http://www.mathopenref.com/circle.html">http://www.mathopenref.com/circle.html</a> (circle definition) <a href="http://www.mathopenref.com/tocs/pointstoc.html">http://www.mathopenref.com/tocs/pointstoc.html</a> (point, lines, and planes definitions)

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<b>Assessment:</b>	Students mastering the concept and skills of this learning experience should be able to answer questions such as: What is the difference between a line and line segment? Will three line segments always make a triangle? Why or why not? What is the connection between parallel and perpendicular lines? Why do we need precise definitions of geometric terms? How does an angle relate to a circle? Can you have an angle that is more than 180 degrees? Why or why not?	
<b>Differentiation:</b> (Multiple means for students to access content and multiple modes for student to express understanding.)	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	<a href="http://wvde.state.wv.us/strategybank/FrayerModel.html">http://wvde.state.wv.us/strategybank/FrayerModel.html</a> (Frayer model examples of mathematical definitions) <a href="http://www.shmoop.com/common-core-standards/ccss-hs-g-co-1.html">http://www.shmoop.com/common-core-standards/ccss-hs-g-co-1.html</a> (definitions and samples questions for G.CO.1)	Students can match critical language to visual examples, non-examples and definitions Students can answer questions that connect definitions for each concept in the learning experience to real world examples
<b>Extensions for depth and complexity:</b>	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	<a href="http://www.illustrativemathematics.org/illustrations/1504">http://www.illustrativemathematics.org/illustrations/1504</a> (discussion on the two definitions of a trapezoid) <a href="http://www.eisd.net/cms/lib04/TX01001208/Centricity/Domain/599/DoubleBubbleMap.pdf">http://www.eisd.net/cms/lib04/TX01001208/Centricity/Domain/599/DoubleBubbleMap.pdf</a> (thinking map for comparing and contrasting)	Students can present to the class the competing definitions for figures such as trapezoids and explain the implications of each definition and explain why precision is necessary in defining geometric terms
<b>Key Knowledge and Skills:</b>	<ul style="list-style-type: none"> <li>Know precise definitions of angle, circle, circular arc, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc</li> </ul>	
<b>Critical Language:</b>	Circle, angle, parallel lines, perpendicular lines, parallelogram, triangle, line, line segment, point, definition, identify, sort, precision	

<b>Learning Experience # 2</b>	
<p>The teacher may provide examples of rigid transformation (reflection, rotation, translation) so that students can explore the impact on coordinates of transforming a figure on a coordinate plane.</p> <p><i>Enactive:</i> Students can rotate, reflect, and translate a nonsymmetrical object (e.g., tetromino) on a coordinate grid.</p> <p><i>Iconic:</i> Students can draw a nonsymmetrical figure on a coordinate grid and then draw the image of the figure after a rotation, reflection, or translation.</p> <p><i>Symbolic:</i> Students can record the initial and image coordinates for the vertices of a figure several basic rotations, reflections and translations.</p>	
<b>Teacher Notes:</b>	Students may need tracing paper, transparencies, mirrors and technology to help visualize each transformation. A nonsymmetrical object can help prevent misconceptions that may naturally occur when trying to transform an object (i.e., rotating a square ninety degrees about the origin may obscure how the vertices have rotated). Rotations are generally the hardest for students to visualize particularly rotations not centered at the origin. Be sure students practice using all four quadrants as a starting point for the transformation. This learning experience is designed to be exploratory to provide comfort with each transformation on the coordinate plane. The next three learning experiences break down each transformation separately to provide students with a deeper exploration.
<b>Generalization Connection(s):</b>	Algebraic representations model geometric transformations performed on a coordinate plane



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<b>Teacher Resources:</b>	<a href="http://mathworld.wolfram.com/Tetromino.html">http://mathworld.wolfram.com/Tetromino.html</a> (examples of tetrominoes) <a href="https://commoncoregeometry.wikispaces.hcps.org/Unit+1">https://commoncoregeometry.wikispaces.hcps.org/Unit+1</a> (transformation resources, including a unit on transformation with real world examples)	
<b>Student Resources:</b>	<a href="http://www.ixl.com/math/geometry">http://www.ixl.com/math/geometry</a> (practice on transformations, L1 through L8) <a href="http://www.shmoop.com/common-core-standards/ccss-hs-g-co-2.html">http://www.shmoop.com/common-core-standards/ccss-hs-g-co-2.html</a> (explanation of the standard and real world sample assessment questions) <a href="http://www.mybookezzz.org/special-education-lesson-plan-for-coordinate-planes/">http://www.mybookezzz.org/special-education-lesson-plan-for-coordinate-planes/</a> (practice with graphing on a coordinate grid)	
<b>Assessment:</b>	Students mastering the concept and skills of this learning experience should be able to answer questions such as: Why do rotations, reflections, and translations preserve the shape and size of a figure? What needs to be included in a precise set of directions for each transformation to allow someone else to perform them? Which transformations create the same images, for example do any rotations and reflections result in the same image? What happens if you combine two transformations? What transformations will carry an object onto itself?	
<b>Differentiation:</b> (Multiple means for students to access content and multiple modes for student to express understanding.)	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	<a href="http://www.mathsisfun.com/geometry/rotation.html">http://www.mathsisfun.com/geometry/rotation.html</a> (simple explanations of each transformation and practice questions)	Students can answer questions about rotations, reflections, and translations on a coordinate grid
<b>Extensions for depth and complexity:</b>	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	<a href="http://www.intmath.com/blog/music-and-transformation-geometry/5074">http://www.intmath.com/blog/music-and-transformation-geometry/5074</a> (music and transformations)	Students can create a short musical piece using the concept of transformations
<b>Key Knowledge and Skills:</b>	<ul style="list-style-type: none"> <li>Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself</li> <li>Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure</li> </ul>	
<b>Critical Language:</b>	Coordinate grid, quadrants, symmetry, y-axis, x-axis, rotation, reflection, translation, rigid transformation, image, original, transformation	

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Learning Experience # 3	
<p>The teacher may provide examples of rotations so that students can begin to create precise geometric and algebraic definitions of a rotation.</p> <p><i>Enactive:</i> Students can rotate a point on a coordinate grid using string. Students can anchor one end of the string at center of rotation and stretch the other side of the string out to the original point. By keeping the string taut students can rotate the string a given number of degrees to find the image of the original point.</p> <p><i>Iconic:</i> Students can draw with a compass the circle of rotation for a point. The center of the circle will be the center of rotation and the radius is the length from the center of rotation to the original point. By drawing the resulting circle, students can find images for any angle of rotation by finding the point that creates an arc on the circle between the original point and its image with the appropriate measure.</p> <p><i>Symbolic:</i> Students can describe a rotation, using angle of rotation, direction (e.g., counterclockwise or clockwise) and center. Students can determine the algebraic generalization on the coordinate plane for angles of rotation about the origin for 90, 180, 270 and 360 degrees rotations (e.g, <math>T(x, y) \rightarrow T'(y, -x)</math> when rotated counterclockwise 270 degrees about the origin on a coordinate grid).</p>	
<b>Teacher Notes:</b>	Rotations can be viewed geometrically as a movement along the arc of a circle and algebraically as the coordinates of an image for the point $(x, y)$ . This learning experience assumes rotations about the origin to provide a consistent look at algebraic representations of rotations. Students can explore rotations with centers beyond the origin and degrees other than 90, 180, 270 and 360 but they should not be expected to generalize the algebraic notation for these other rotations. It is important emphasize the connection between the definitions for a circle and angles as a way of understanding a rotation.
<b>Generalization Connection(s):</b>	Precise definitions of basic geometric concepts facilitate the development of careful proofs Algebraic representations model geometric transformations performed on a coordinate plane
<b>Teacher Resources:</b>	<p><a href="https://www.khanacademy.org/math/geometry/transformations">https://www.khanacademy.org/math/geometry/transformations</a> (videos explaining each type of transformation on a coordinate grid)</p> <p><a href="http://www.regentsprep.org/Regents/math/geometry/GT5/reviewtransformations.htm">http://www.regentsprep.org/Regents/math/geometry/GT5/reviewtransformations.htm</a> (review of transformation notations, formulas and definitions)</p>
<b>Student Resources:</b>	<p><a href="http://www.mathwarehouse.com/transformations/rotations-in-math.php">http://www.mathwarehouse.com/transformations/rotations-in-math.php</a> (interactive demonstration of how to perform a rotation in math)</p> <p><a href="http://www.ixl.com/math/geometry/rotations-find-the-coordinates">http://www.ixl.com/math/geometry/rotations-find-the-coordinates</a> (practice writing coordinates for rotations around the center)</p>
<b>Assessment:</b>	<p>Students mastering the concept and skills of this learning experience should be able to answer questions such as:</p> <p>How can you determine the angle of rotation when provided with two points and the point of rotation?</p> <p>How can you define a rotation using the concept of a circle?</p> <p>What happens if the center of rotation is the same as the point being rotated?</p> <p>What are coordinates of the image of a point <math>(x, y)</math> rotated about the origin 90, 180, 270, 360 degrees? Why does it not matter if the original point is in the 3<sup>rd</sup> quadrant?</p>

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<b>Differentiation:</b> (Multiple means for students to access content and multiple modes for student to express understanding.)	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	<a href="http://enlvm.usu.edu/ma/nav/activity.jsp?sid=shared&amp;cid=emready@transformations&amp;lid=32">http://enlvm.usu.edu/ma/nav/activity.jsp?sid=shared&amp;cid=emready@transformations&amp;lid=32</a> (visual images for each transformation) <a href="http://learnzillion.com/lessons/2633-measure-full-and-half-rotations">http://learnzillion.com/lessons/2633-measure-full-and-half-rotations</a> (resources for reviewing ideas about angles and circles) <a href="http://www.gradeamathhelp.com/transformation-geometry.html">http://www.gradeamathhelp.com/transformation-geometry.html</a> (hints about each transformation)	<p>Students can develop fluency with the definitions of a rotation scaffolded by visual images of the vocabulary</p> <p>Students can measure angles along the arc of a circle by first reviewing these ideas in a video as prerequisite understanding and skills for this learning experience</p> <p>Students can answer questions on rotations to reinforce their understanding of rotations</p>
<b>Extensions for depth and complexity:</b>	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	N/A	Students can use the distance formula to determine the location of a point after a rotation when the point of rotation is not at the center or for degrees other than 90, 180, 270, and 360 degrees
<b>Key Knowledge and Skills:</b>	<ul style="list-style-type: none"> <li>Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments</li> </ul>	
<b>Critical Language:</b>	Circle, arc, origin, direction, clockwise, counterclockwise, degrees, point of rotation, coordinates, original, image, coordinate grid, quadrants, compass, algebraic generalization, transformation	

<b>Learning Experience # 4</b>	
<p>The teacher may provide examples of reflections so that students can begin to create precise geometric and algebraic definitions of a reflection.</p> <p><i>Enactive:</i> Students can be given two points and asked to find the line of reflection by folding and then a point and line of reflection and asked to find the image of the point.</p> <p><i>Iconic:</i> Students can be given two points and asked to draw the line of reflection (i.e., the perpendicular bisector of the two points) using either geometry software, geometric construction tools, or a protractor and ruler.</p> <p><i>Symbolic:</i> Students can describe a rotation using the line of reflection. Students can determine the algebraic generalization of a reflection on the coordinate plane (e.g., <math>T(x, y) \rightarrow T'(x, -y)</math> when reflected over the x-axis on a coordinate grid).</p>	
<b>Teacher Notes:</b>	Reflections can be viewed geometrically as the image, which creates a perpendicular bisector between the line of reflection and the original point, and algebraically as the coordinates of an image for the point $(x, y)$ . Students can be given opportunities to see that the line of reflection is perpendicular to the segment between the two reflected points. Students might also notice that reflecting a point across a line twice will result in the point carrying onto itself. Students can also connect rotations of 180 degrees to reflections.
<b>Generalization Connection(s):</b>	Precise definitions of basic geometric concepts facilitate the development of careful proofs Algebraic representations model geometric transformations performed on a coordinate plane

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<b>Teacher Resources:</b>	<a href="https://www.khanacademy.org/math/geometry/transformations">https://www.khanacademy.org/math/geometry/transformations</a> (videos explaining each type of transformation on a coordinate grid) <a href="http://www.youtube.com/watch?v=EDIMOFFc4J0&amp;feature=related">http://www.youtube.com/watch?v=EDIMOFFc4J0&amp;feature=related</a> (video about reflections) <a href="http://www.regentsprep.org/Regents/math/geometry/GT5/reviewtranformations.htm">http://www.regentsprep.org/Regents/math/geometry/GT5/reviewtranformations.htm</a> (review of transformation notations, formulas and definitions)	
<b>Student Resources:</b>	<a href="http://www.mathwarehouse.com/transformations/reflections-in-math.php">http://www.mathwarehouse.com/transformations/reflections-in-math.php</a> (activity to examine coordinates) <a href="http://www.ixl.com/math/geometry/reflections-find-the-coordinates">http://www.ixl.com/math/geometry/reflections-find-the-coordinates</a> (applet to practice writing coordinate points after a reflection)	
<b>Assessment:</b>	Students mastering the concept and skills of this learning experience should be able to answer questions such as: What rotation is equivalent to a reflection? What happens when the line of reflection goes through point? How do you find a line of reflection given two points? Why is the perpendicular bisector the same as the line of reflection of two points? How can you define a reflection using the concept of a perpendicular bisector?	
<b>Differentiation:</b> (Multiple means for students to access content and multiple modes for student to express understanding.)	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	<a href="http://www.gradeamathhelp.com/transformation-geometry.html">http://www.gradeamathhelp.com/transformation-geometry.html</a> (visual images for each transformation) <a href="http://enlvm.usu.edu/ma/nav/activity.jsp?sid=shared&amp;cid=emready@transformations&amp;lid=51">http://enlvm.usu.edu/ma/nav/activity.jsp?sid= shared&amp;cid=emready@transformations&amp;lid=51</a> (practice on reflections)	Students can develop fluency with the definitions of a rotation scaffolded by visual images of the vocabulary Students can answer questions on this applet to reinforce their understanding of reflections
<b>Extensions for depth and complexity:</b>	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	<a href="http://www.etymonline.com/index.php?term=reflection">http://www.etymonline.com/index.php?term=reflection</a> (entomology of the word reflection)	Students can create metaphors for reflection and connect its mathematical uses to other content areas
<b>Key Knowledge and Skills:</b>	<ul style="list-style-type: none"> <li>Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments</li> </ul>	
<b>Critical Language:</b>	Reflection, line segment, perpendicular, bisect, x-axis, y-axis, $y=x$ , line of reflection, coordinates, original, image, coordinate grid, quadrants, algebraic generalization, transformation	

<b>Learning Experience # 5</b>
<p>The teacher may provide examples of translations so that students can begin to create precise geometric and algebraic definitions of a translation.</p> <p><i>Enactive:</i> Students can move on a life size coordinate grid given directions for a translation such as move -2 units vertically or <math>(x, y-2)</math>.</p> <p><i>Iconic:</i> Students can draw a translated a point on a coordinate grid given directions for a translation.</p> <p><i>Symbolic:</i> Students can describe a translation using verbal and algebraic descriptions given two points.</p>

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<b>Teacher Notes:</b>	Translations can be viewed geometrically as moving an original point a particular distance and direction (e.g., vector) and algebraically as the coordinates of an image for the point $(x, y)$ . Students may struggle to fluently move a point both negatively and positively in vertical and horizontal directions and to coordinate the movement for both the $x$ and $y$ coordinates (e.g. $T(x, y) \rightarrow T'(x+2, y-3)$ translates a point two units to the right and three units down on a coordinate grid).	
<b>Generalization Connection(s):</b>	Precise definitions of basic geometric concepts facilitate the development of careful proofs Algebraic representations model geometric transformations performed on a coordinate plane	
<b>Teacher Resources:</b>	<a href="https://www.khanacademy.org/math/geometry/transformations">https://www.khanacademy.org/math/geometry/transformations</a> (videos explaining each type of transformation on a coordinate grid) <a href="http://www.regentsprep.org/Regents/math/geometry/GT5/reviewtransformations.htm">http://www.regentsprep.org/Regents/math/geometry/GT5/reviewtransformations.htm</a> (review of transformation notations, formulas and definitions)	
<b>Student Resources:</b>	<a href="http://www.mathsisfun.com/geometry/translation.html">http://www.mathsisfun.com/geometry/translation.html</a> (translate objects while examining changes in coordinates) <a href="http://www.ixl.com/math/geometry/translations-write-the-rule">http://www.ixl.com/math/geometry/translations-write-the-rule</a> (writing the rule for a translation)	
<b>Assessment:</b>	Students mastering the concept and skills of this learning experience should be able to answer questions such as: How can you describe a translation that will carry a point back to itself? When translating several points how are the lines connecting the points to their images related? How are parallel lines related to translations? How can you find a translation given two points? How can you define a translation geometrically and algebraically?	
<b>Differentiation:</b> (Multiple means for students to access content and multiple modes for student to express understanding.)	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	<a href="http://www.gradeamathhelp.com/transformation-geometry.html">http://www.gradeamathhelp.com/transformation-geometry.html</a> (visual images for each transformation) <a href="http://enlvm.usu.edu/ma/nav/activity.jsp?sid=shared&amp;cid=emready@transformations&amp;lid=29">http://enlvm.usu.edu/ma/nav/activity.jsp?sid=shared&amp;cid=emready@transformations&amp;lid=29</a> (practice on translations)	Students can develop fluency with the definitions of a rotation scaffolded by visual images of the vocabulary Students can answer practice questions to reinforce their understanding of translations
<b>Extensions for depth and complexity:</b>	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	<a href="http://enlvm.usu.edu/ma/nav/activity.jsp?sid=shared&amp;cid=emready@transformations&amp;lid=61&amp;aid=940049925">http://enlvm.usu.edu/ma/nav/activity.jsp?sid=shared&amp;cid=emready@transformations&amp;lid=61&amp;aid=940049925</a> (relating translations to vectors)	Students can create a presentation for the class on the use of vectors to describe translations and their application to real world situation
<b>Key Knowledge and Skills:</b>	<ul style="list-style-type: none"> <li>Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments</li> </ul>	
<b>Critical Language:</b>	Translations, vertically, horizontally, image, original, parallel, coordinates, coordinate grid, quadrants, algebraic generalization, transformation	

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Learning Experience # 6		
The teacher may provide students with examples of all three types of transformations so that students can gain fluency in describing transformations verbally, algebraically, and graphically.		
<b>Generalization Connection(s):</b>	Precise definitions of basic geometric concepts facilitate the development of careful proofs Algebraic representations model geometric transformations performed on a coordinate plane	
<b>Teacher Resources:</b>	<a href="http://map.mathshell.org/materials/lessons.php?taskid=524&amp;subpage=concept">http://map.mathshell.org/materials/lessons.php?taskid=524&amp;subpage=concept</a> (formative lesson with cards students can complete showing transformations graphically, algebraically and graphically)	
<b>Student Resources:</b>	N/A	
<b>Assessment:</b>	Students mastering the concept and skills of this learning experience should be able to answer questions such as: How can you describe a transformation algebraically? What does $(x, y) \rightarrow (y, x)$ mean? What transformation does it define? How are translations, rotations, and reflections similar and different? Which transformation is the easiest to visualize? Which is the hardest? Why?	
<b>Differentiation:</b> (Multiple means for students to access content and multiple modes for student to express understanding.)	Access (Resources and/or Process)	Expression (Products and/or Performance)
	<a href="http://www.regentsprep.org/Regents/math/geometry/GT5/reviewtransformations.htm">http://www.regentsprep.org/Regents/math/geometry/GT5/reviewtransformations.htm</a> (review of transformation notations, formulas and definitions) <a href="http://illuminations.nctm.org/Lesson.aspx?id=3704">http://illuminations.nctm.org/Lesson.aspx?id=3704</a> (lesson plan for using an online applet that practices all three transformations)	Students can complete the algebraic and verbal descriptions of a transformation using scaffolding on the notation and definitions for each transformation Students can complete practice questions for all three transformations
<b>Extensions for depth and complexity:</b>	Access (Resources and/or Process)	Expression (Products and/or Performance)
	<a href="http://www.discoveryeducation.com/teachers/free-lesson-plans/discovering-math-exploring-geometry.cfm">http://www.discoveryeducation.com/teachers/free-lesson-plans/discovering-math-exploring-geometry.cfm</a> (unit on creating a three dimensional model of a city)	Students can create a three-dimensional model of a city using transformations
<b>Key Knowledge and Skills:</b>	<ul style="list-style-type: none"> <li>Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments</li> </ul>	
<b>Critical Language:</b>	Verbally, algebraically, graphically, translations, vertically, horizontally, reflection, line segment, perpendicular, bisect, x-axis, y-axis, $y=x$ , line of reflection, circle, arc, origin, direction, clockwise, counterclockwise, degrees, point of rotation, image, original, parallel, coordinates, coordinate grid, quadrants, algebraic generalization, transformation	

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<b>Learning Experience # 7</b>	
<p>The teacher may provide examples of transformations (including those that do not preserve distances and angles in addition to examples of reflections, rotations, and translations) so that students can explore which types of transformation preserves distances and angles (i.e., creates congruent figures).</p> <p><i>Enactive:</i> Students can explore reflections and rotations on rectangles, parallelograms, trapezoids, and regular polygons to find rotations and reflections that carry each type of figure onto itself.</p> <p><i>Ionic:</i> Students can draw the image of a figure when provided with algebraic notation of a transformation and identify if the transformation is a reflection, translation, or rotation. Students can justify if the translation created a congruent figure by determining if distances and angles were preserved.</p> <p><i>Symbolic:</i> Students can create an argument that each type of rigid transformation is a function based on the precise definition for each rigid transformation (reflections, translations, and rotations) and the definition of a function.</p>	
<b>Teacher Notes:</b>	<p>This is the first learning experience where students will encounter non-rigid transformations such as <math>T(x, y) \rightarrow T'(2x, 2y)</math> (i.e., a dilation) or <math>T(x, y) \rightarrow T'(-2x, 3y)</math>. It may be helpful for students to reason through why dilations are not functions because each point in the original is not mapped to one and only one point in the image before trying to show why reflections, rotations, and translations are functions. Throughout this learning experience students can be introduced to the notation for congruent angles and sides on geometric drawings. Before the end of this unit the teacher may also introduce the notations for perpendicular and parallel lines.</p>
<b>Generalization Connection(s):</b>	<p>Precise definitions of basic geometric concepts facilitate the development of careful proofs                      Algebraic representations model geometric transformations performed on a coordinate plane                      Sequences of transformations or combinations of angles and side lengths can determine the congruence of shapes</p>
<b>Teacher Resources:</b>	<p><a href="http://www.whyslopes.com/5080More_Algebra/54004_Functions/00505_Function_notation_for_geometric_transformations.html">http://www.whyslopes.com/5080More_Algebra/54004_Functions/00505_Function_notation_for_geometric_transformations.html</a> (function notation for transformations)</p> <p><a href="https://docs.google.com/a/sjsu.edu/file/d/0BvBN4IxmWgMTBjYTZjMiEtNGFkOC00MzUxLWJhZGUtMDM2MmQyOWFIN2M0/edit?hl=en&amp;pli=1">https://docs.google.com/a/sjsu.edu/file/d/0BvBN4IxmWgMTBjYTZjMiEtNGFkOC00MzUxLWJhZGUtMDM2MmQyOWFIN2M0/edit?hl=en&amp;pli=1</a> (precise definitions for transformations)</p> <p><a href="http://www.ixl.com/math/geometry">http://www.ixl.com/math/geometry</a> (choose L.11, transformations that carry a polygon onto itself, and L.9, classify congruence transformations)</p> <p><a href="http://www.mathematicsvisionproject.org/uploads/1/1/6/3/11636986/sec1_mod5_ccp_tn_112612.pdf">http://www.mathematicsvisionproject.org/uploads/1/1/6/3/11636986/sec1_mod5_ccp_tn_112612.pdf</a> (unit exploring congruence and transformations)</p> <p><a href="https://commoncoregeometry.wikispaces.hcps.org/Unit+1">https://commoncoregeometry.wikispaces.hcps.org/Unit+1</a> (congruence as rigid motions)</p>
<b>Student Resources:</b>	<p><a href="http://caccssm.cmpso.org/a/cmpso.org/caccss-resources/geometry-task-force/geometry-resources/isometry-unit">http://caccssm.cmpso.org/a/cmpso.org/caccss-resources/geometry-task-force/geometry-resources/isometry-unit</a> (section A8 algebraic transformations: gives a list of transformations written algebraically so students can draw the resulting figure on a coordinate grid and determine what type of transformation was used)</p>
<b>Assessment:</b>	<p>Students mastering the concept and skills of this learning experience should be able to answer questions such as:                      Which transformations create congruent figures and why?                      Why are reflections, translations, and rotations called rigid transformations?                      Why are rigid transformations functions?                      Could transformations be a function and not create a congruent figure? Why or why not?                      Which rigid transformations can carry a figure onto itself (i.e., the identify transformation)? Does the figure being transformed make a difference?</p>

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<b>Differentiation:</b> (Multiple means for students to access content and multiple modes for student to express understanding.)	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	<a href="http://www.worksheetworks.com/miscellanea/graphic-organizers.html">http://www.worksheetworks.com/miscellanea/graphic-organizers.html</a> (definitions of each transformation and a function using the Frayer model)	Students can provide a written explanation of why each rigid transformation is a function scaffolded by definitions for transformations and a function
<b>Extensions for depth and complexity:</b>	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	<a href="http://en.wikipedia.org/wiki/Inversive_geometry">http://en.wikipedia.org/wiki/Inversive_geometry</a> (describes the definition of geometric inverse)	Students can write the inverse transformation for each of the transformations in this learning experience
<b>Key Knowledge and Skills:</b>	<ul style="list-style-type: none"> <li>• Represent transformations in the plane and describe transformations as functions that take points in the plane as inputs and give other points as outputs</li> <li>• Compare transformations that preserve distance and angle to those that do not</li> <li>• Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself</li> <li>• Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent</li> </ul>	
<b>Critical Language:</b>	Congruence, function, distances, angles, parallelogram, trapezoid, regular polygon, precise, definition, rotation, reflection, translation, rigid transformation	

**Learning Experience # 8**

The teacher may provide students with pairs of figures on a coordinate grid (only some of which are congruent) so that students can use transformations to determine congruency.

*Enactive:* Students can use tracing paper to show a sequence of transformations to map one figure onto another.

*Symbolic:* Students can describe algebraically the sequence of transformations necessary to map one figure onto another to show congruence.

<b>Teacher Notes:</b>	The enactive stage should support students in the symbolic stage as they describe a sequence of transformations. This is the first time students are composing transformations (i.e., linking transformations). Students should test out whether the order matters when specifying a sequence of transformations.
<b>Generalization Connection(s):</b>	Sequences of transformations or combinations of angles and side lengths can determine the congruence of shapes
<b>Teacher Resources:</b>	<a href="http://caccssm.cmpso.org/a/cmpso.org/caccss-resources/geometry-task-force/geometry-resources/isometry-unit">http://caccssm.cmpso.org/a/cmpso.org/caccss-resources/geometry-task-force/geometry-resources/isometry-unit</a> (A6 congruence activity: given two figures describe a sequence of transformations to map one figure onto another) <a href="https://commoncoregeometry.wikispaces.hcpss.org/Unit+1">https://commoncoregeometry.wikispaces.hcpss.org/Unit+1</a> (congruence as rigid motions) <a href="http://www.showme.com/sh/?h=fPKkK7E">http://www.showme.com/sh/?h=fPKkK7E</a> (congruency and transformations)
<b>Student Resources:</b>	N/A
<b>Assessment:</b>	Students mastering the concept and skills of this learning experience should be able to answer questions such as: How can you tell if two figures are congruent by using transformations? Is there more than one sequence of transformations that can be used to map a figure to its image? When does the order of transformations matter when determining the sequence of transformations that map a figure onto its image?



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<b>Differentiation:</b> (Multiple means for students to access content and multiple modes for student to express understanding.)	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	Teachers may provide students with the transformations used to create image of a figure	Students can sequence the transformation options provided by the teacher to determine if the two figures are congruent
<b>Extensions for depth and complexity:</b>	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	<a href="http://www.mcescher.com/">http://www.mcescher.com/</a> (official McEscher website with examples of his art)	Students can identify the types of transformations used to create art created by McEscher Students can create their own artwork inspired by McEscher Students can create sequences of transformations that are commutative (i.e., a composition of transformations for which order does not matter)
<b>Key Knowledge and Skills:</b>	<ul style="list-style-type: none"> <li>• Represent transformations in the plane and describe transformations as functions that take points in the plane as inputs and give other points as outputs</li> <li>• Compare transformations that preserve distance and angle to those that do not</li> <li>• Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself</li> <li>• Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent</li> <li>• Specify a sequence of transformations that will carry a given figure onto another</li> </ul>	
<b>Critical Language:</b>	Sequence, order, congruent, function, distances, angles, rotation, reflection, translation, rigid transformation	

<b>Learning Experience # 9</b>	
<p>The teacher may provide students with attributes of triangles (e.g., one angle and one side) so that students can explore which attributes are necessary to ensure congruent triangles.</p> <p><i>Enactive:</i> Students can use straight manipulatives (e.g., toothpicks, straws) to determine if there is more than one triangle possible from a list of attributes.</p> <p><i>Iconic:</i> Students can draw triangles with particular attributes to determine if there is more than one triangle possible from a list of attributes.</p> <p><i>Symbolic:</i> Students will provide an informal justification of whether the attributes will always result in congruent triangles and critique the reasoning of others.</p>	
<b>Teacher Notes:</b>	Students may struggle correctly ordering the sides and angles when doing determining congruent triangles. It is sometimes difficult for them to determine if the side is between the angles or not. This learning experience is designed to provide an informal experience with these ideas. The next learning experience formalizes these ideas by proving the SSS, SAS, and ASA theorems.
<b>Generalization Connection(s):</b>	Precise definitions of basic geometric concepts facilitate the development of careful proofs Sequences of transformations or combinations of angles and side lengths can determine the congruence of shapes
<b>Teacher Resources:</b>	<a href="http://map.mathshell.org/materials/lessons.php?taskid=452&amp;subpage=concept">http://map.mathshell.org/materials/lessons.php?taskid=452&amp;subpage=concept</a> (lesson for analyzing which attributes of a triangles are necessary to prove congruence)
<b>Student Resources:</b>	N/A

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<b>Assessment:</b>	Students mastering the concept and skills of this learning experience should be able to answer questions such as: What attributes do you need to conclude if two triangles are congruent? Why is it not always sufficient to know two angles and a side to determine congruency in triangles? Why is it not always sufficient to know any two sides and an angle to determining congruency in triangles? What are some characteristics of a quality proof?	
<b>Differentiation:</b> (Multiple means for students to access content and multiple modes for student to express understanding.)	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	<a href="http://map.mathshell.org/materials/lessons.php?taskid=452&amp;subpage=concept">http://map.mathshell.org/materials/lessons.php?taskid=452&amp;subpage=concept</a> (lesson for analyzing which attributes of a triangles are necessary to prove congruence)	Students can verbally explain to a partner only four of the nine examples in the learning experience (SSS, SAS, ASA and a non-example) prior to working as a team to write their informal reasoning
<b>Extensions for depth and complexity:</b>	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	<a href="http://www.geometrycommoncore.com/content/unit1/gco8/gco8.html">http://www.geometrycommoncore.com/content/unit1/gco8/gco8.html</a> (resource for examining all the different ways of looking at triangle congruence)	Students can create an informal proof for the more specialized cases of triangle congruence such as for AAS and HL
	<ul style="list-style-type: none"> <li>Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent</li> </ul>	
<b>Critical Language:</b>	Triangles, angles, sides, congruence, attributes, necessary, sufficient, critique, corresponding	

<b>Learning Experience # 10</b>	
The teacher may provide diagrams of congruent triangles so that students can begin to formalize their understanding of triangle congruence developed in the previous learning experience.	
<b>Teacher Notes:</b>	Students struggle to understand the need for proof in geometry. Drawings accompanying proofs provide visual evidence of what the problem is trying to prove and students sometimes see little need to prove what a drawing shows. Students should be encouraged to understand a drawing is not a proof because it is not generalizable and can be deceiving.
<b>Generalization Connection(s):</b>	Precise definitions of basic geometric concepts facilitate the development of careful proofs Sequences of transformations or combinations of angles and side lengths can determine the congruence of shapes
<b>Teacher Resources:</b>	<a href="http://www.geometrycommoncore.com/content/unit1/gco8/gco8.html">http://www.geometrycommoncore.com/content/unit1/gco8/gco8.html</a> (teaching resources for proving triangle congruence, you can see the resources for free and pay a nominal fee to download everything on the site) <a href="http://www.illustrativemathematics.org/illustrations/109">http://www.illustrativemathematics.org/illustrations/109</a> (proof of SAS using reflections) <a href="https://commoncoregeometry.wikispaces.hcpss.org/Unit+1">https://commoncoregeometry.wikispaces.hcpss.org/Unit+1</a> (proving geometric theorems with video)
<b>Student Resources:</b>	<a href="http://www.shmoop.com/common-core-standards/ccss-hs-g-co-8.html">http://www.shmoop.com/common-core-standards/ccss-hs-g-co-8.html</a> (practice questions about congruent triangles)

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<b>Assessment:</b>	Students mastering the concept and skills of this learning experience should be able to answer questions such as: How do parallel lines help prove congruent triangles? Why is SAS a congruency theorem but SSA isn't? Why is proof important in geometry?	
<b>Differentiation:</b> (Multiple means for students to access content and multiple modes for student to express understanding.)	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	<a href="http://www.ixl.com/math/geometry">http://www.ixl.com/math/geometry</a> (congruent triangle proofs are provided with scaffolding)	Students can by provide missing statements and reasons to complete unfinished portion of proofs
<b>Extensions for depth and complexity:</b>	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	N/A	Students can construct proofs for determining the necessary and sufficient attributes for determining congruency of other shapes (e.g., parallelograms, regular polygons)
<b>Key Knowledge and Skills:</b>	<ul style="list-style-type: none"> <li>• Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent</li> <li>• Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions</li> </ul>	
<b>Critical Language:</b>	Proof, theorem, congruent, triangles, angles, sides, congruence, attributes, necessary, sufficient, corresponding	

<b>Learning Experience # 11</b>	
<p>The teacher may pose parallel line scenarios so that students can explore angle relationships of parallel lines cut by a transversal and related proofs.</p> <p><i>Enactive:</i> Students can investigate relationships between angles by exploring translated parallelograms (i.e., tiled parallelograms) using tracing paper or transparencies. Students can then color all the congruent acute angles in one color and the congruent obtuse angles in another color.</p> <p><i>Iconic:</i> Students can identify the key angle relationships (e.g., alternate, corresponding, vertical) on a portion of the tiled parallelograms (e.g., a set of parallel lines cut by a transversal) and determine which relationships are congruent and which are supplementary and explain these to another student by using rotations, reflections and translations.</p> <p><i>Symbolic:</i> Students can prove the angle relationships created when a set of parallel lines is cut by a transversal.</p>	
<b>Generalization Connection(s):</b>	Precise definitions of basic geometric concepts facilitate the development of careful proofs
<b>Teacher Resources:</b>	<a href="http://www.shmoop.com/common-core-standards/ccss-hs-g-co-9.html">http://www.shmoop.com/common-core-standards/ccss-hs-g-co-9.html</a> (detailing the standard, provides assessment questions) <a href="http://learni.st/users/S33572/boards/2805-proving-theorems-about-lines-and-angles-common-core-standard-9-12-g-co-9">http://learni.st/users/S33572/boards/2805-proving-theorems-about-lines-and-angles-common-core-standard-9-12-g-co-9</a> (examples of the angle relationships, videos about the relationships and example proofs) <a href="http://www.brightstorm.com/math/precalculus/equations-of-lines-parabolas-circles/converse-of-parallel-lines-theorem/">http://www.brightstorm.com/math/precalculus/equations-of-lines-parabolas-circles/converse-of-parallel-lines-theorem/</a> (video about angle theorems) <a href="http://www.cpm.org/teachers/resourcesCCG.htm">http://www.cpm.org/teachers/resourcesCCG.htm</a> (select Core Connections Geometry, Chapter 2, Resource page 2.1.2 for a tiling of parallelograms) <a href="http://www.cpm.org/technology/CCG/">http://www.cpm.org/technology/CCG/</a> (select 2.1.2 - video of the translation of a parallelogram into the tiling pattern)

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<b>Student Resources:</b>	<a href="http://www.ixl.com/math/geometry">http://www.ixl.com/math/geometry</a> (Use D.2, D.4 and D.5. Identifying angle relationships and proving angle relationship theorems) <a href="http://www.regentsprep.org/Regents/math/geometry/GP8/Lparallel.htm">http://www.regentsprep.org/Regents/math/geometry/GP8/Lparallel.htm</a> (easy to read examples of angle theorems) <a href="http://quizlet.com/9815961/flashcards">http://quizlet.com/9815961/flashcards</a> (flashcards of definitions and theorems related to parallel lines)	
<b>Assessment:</b>	Students mastering the concept and skills of this lesson should be able to answer questions such as: Why are pairs of alternate interior (exterior) angles congruent? Why are pairs of corresponding angles supplementary? Why are all the angles, created by parallel lines cut by transversal, either congruent or supplementary? What is the difference between a proof and just explaining your reasoning?	
<b>Differentiation:</b> (Multiple means for students to access content and multiple modes for student to express understanding.)	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	<a href="http://img.docstoccdn.com/thumb/orig/122161993.png">http://img.docstoccdn.com/thumb/orig/122161993.png</a> (handout with angle relationships) <a href="http://www.feromax.com/cgi-bin/Provelt.pl?task=getproofslist">http://www.feromax.com/cgi-bin/Provelt.pl?task=getproofslist</a> (proofs of angle relationships with scaffolding)	Students can identify the types of angles on the tiled parallelograms using a handout showing the angle relationships Students can complete a two-column proof with scaffolding by correctly ordering a list of statements and their corresponding reasons
<b>Extensions for depth and complexity:</b>	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	<a href="http://www.illustrativemathematics.org/illustrations/56">http://www.illustrativemathematics.org/illustrations/56</a> (example of a complex parallel line problem)	Students can design unique angle measure problems for parallel lines
<b>Key Knowledge and Skills:</b>	<ul style="list-style-type: none"> <li>Prove theorems about lines and angles, triangles, and parallelograms</li> </ul>	
<b>Critical Language:</b>	Congruent, transversal, parallel lines, linear pair, vertical angles, alternate interior angles, alternate exterior angles, corresponding angles, supplementary, vertex, proof, parallelograms, translations, reflections, rotations, transformations	

<b>Learning Experience # 12</b>	
<p>The teacher may provide opportunities to investigate parallelograms so that students can explore precise mathematical proofs for properties of parallelograms.</p> <p><i>Enactive:</i> Students can create hypotheses about properties of parallelograms by using transformations (i.e., by folding or tracing) to compare sides, angles and diagonals of parallelograms including those of special parallelograms such as rectangles, squares, and rhombi.</p> <p><i>Iconic:</i> Students can test their hypotheses about properties of parallelograms by using geometric software.</p> <p><i>Symbolic:</i> Students can prove theorems about properties of parallelograms.</p>	
<b>Teacher Notes:</b>	Proofs about properties of parallelograms should include: opposite sides are congruent; opposite angles are congruent; the diagonals of a parallelogram bisect each other; and conversely, rectangles are parallelograms with congruent diagonals. Students may struggle to make the connection between the special angle relationships formed by parallel lines cut by transversals and the opposite angles in a parallelogram. It may be helpful for students to make connections to the translated parallelograms (e.g., tiled parallelograms) from the previous learning experience.
<b>Generalization Connection(s):</b>	Precise definitions of basic geometric concepts facilitate the development of careful proofs Algebraic representations model geometric transformations performed on a coordinate plane

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<b>Teacher Resources:</b>	<a href="http://mathbits.com/MathBits/GSP/Quadrilaterals.htm">http://mathbits.com/MathBits/GSP/Quadrilaterals.htm</a> (exploration of quadrilaterals using Geometer’s Sketch Pad) <a href="http://www.wyzant.com/resources/lessons/math/geometry/quadrilaterals/proving_parallelograms">http://www.wyzant.com/resources/lessons/math/geometry/quadrilaterals/proving_parallelograms</a> (examples of parallelogram proofs) <a href="http://www.curtiscenter.math.ucla.edu/downloads/Olson.pdf">http://www.curtiscenter.math.ucla.edu/downloads/Olson.pdf</a> (PowerPoint examining proofs related to this unit)	
<b>Student Resources:</b>	<a href="http://www.ixl.com/math/geometry">http://www.ixl.com/math/geometry</a> (practice identifying properties of quadrilaterals and proofs – not all proofs are about parallelograms, sections N.2, N.3, N.4, N.9 and N.10) <a href="http://www.shmoop.com/common-core-standards/ccss-hs-g-co-11.html">http://www.shmoop.com/common-core-standards/ccss-hs-g-co-11.html</a> (explanations and sample questions for G.CO.11)	
<b>Assessment:</b>	Students mastering the concept and skills of this learning experience should be able to answer questions such as: Why are the diagonals of a rectangle congruent? Why are the diagonals of a rhombus perpendicular bisectors? How do definitions contribute to the development of a proof? How do the angle relationships of parallel lines and triangle congruence support proofs about parallelograms?	
<b>Differentiation:</b> (Multiple means for students to access content and multiple modes for student to express understanding.)	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	<a href="http://www.regentsprep.org/Regents/math/geometry/GP9/LParallelogram.htm">http://www.regentsprep.org/Regents/math/geometry/GP9/LParallelogram.htm</a> (completed proofs of parallelogram relationships)	Students can complete proofs with scaffolding such as providing some of the statements and/or reasons and supplying missing information
<b>Extensions for depth and complexity:</b>	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	<a href="http://www.mathopenref.com/coordparallelogram.html">http://www.mathopenref.com/coordparallelogram.html</a> (examines parallelogram properties using coordinate geometry)	Students can justify the properties of parallelograms using coordinate proofs involving slope and the distance formula
<b>Key Knowledge and Skills:</b>	<ul style="list-style-type: none"> <li>Prove theorems about lines and angles, triangles, and parallelograms</li> </ul>	
<b>Critical Language:</b>	Parallelogram, rectangle, square, rhombus, opposite, sides, angles, congruent, bisector, parallel, perpendicular, perpendicular bisector, line segment, equidistant, diagonals, converses, reflections, rotations, translations, transformations	

<b>Learning Experience # 13</b>	
<p>The teacher may provide opportunities to investigate triangles so that students can explore precise mathematical proofs for properties of triangles.</p> <p><i>Enactive:</i> Students can create hypotheses about properties of triangles by using transformations (i.e., by folding or tracing) to compare sides, angles and medians.</p> <p><i>Iconic:</i> Students can test their hypotheses about properties of triangles by using geometric software.</p> <p><i>Symbolic:</i> Students can prove theorems about properties of triangles.</p>	
<b>Teacher Notes:</b>	Proofs about properties of triangles should include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
<b>Generalization Connection(s):</b>	Precise definitions of basic geometric concepts facilitate the development of careful proofs Algebraic representations model geometric transformations performed on a coordinate plane

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<b>Teacher Resources:</b>	<a href="http://teachinginspecialeducation.blogspot.com/2013/05/angle-sum-theorem.html">http://teachinginspecialeducation.blogspot.com/2013/05/angle-sum-theorem.html</a> (example of tearing off the corner of triangle activity) <a href="http://www.mathwarehouse.com/geometry/triangles/">http://www.mathwarehouse.com/geometry/triangles/</a> (examples of triangle proofs)	
<b>Student Resources:</b>	<a href="http://www.regentsprep.org/Regents/math/geometry/GP6/Lisosceles.htm">http://www.regentsprep.org/Regents/math/geometry/GP6/Lisosceles.htm</a> (isosceles triangle resources) <a href="http://www.ixl.com/math/geometry">http://www.ixl.com/math/geometry</a> (Use M.1 and M.7. good practice with triangle mid-segment theorems and proofs.) <a href="http://www.mathsisfun.com/proof180deg.html">http://www.mathsisfun.com/proof180deg.html</a> (example of proofs for triangle angle sum)	
<b>Assessment:</b>	Students mastering the concept and skills of this learning experience should be able to answer questions such as: What is the relationship in all the angles of a triangle? Why must triangle mid-segments be parallel to their bases?	
<b>Differentiation:</b> (Multiple means for students to access content and multiple modes for student to express understanding.)	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	<a href="http://www.regentsprep.org/Regents/math/geometry/GP6/Lisosceles.htm">http://www.regentsprep.org/Regents/math/geometry/GP6/Lisosceles.htm</a> (completed proofs of triangle relationships) <a href="http://www.mathopenref.com/triangle.html">http://www.mathopenref.com/triangle.html</a> (critical language related to triangles)	Students can complete proofs with scaffolding such as providing some of the statements and/or reasons and supplying missing information
<b>Extensions for depth and complexity:</b>	<b>Access</b> (Resources and/or Process)	<b>Expression</b> (Products and/or Performance)
	<a href="http://www.mathopenref.com/coordparallelogram.html">http://www.mathopenref.com/coordparallelogram.html</a> (examines parallelogram properties using coordinate geometry but the ideas can be applied to triangles and there links at the bottom of the page to support this extension)	Students can justify the properties of triangles using coordinate proofs involving slope and the distance formula
<b>Key Knowledge and Skills:</b>	<ul style="list-style-type: none"> <li>• Prove theorems about lines and angles, triangles, and parallelograms</li> </ul>	
<b>Critical Language:</b>	Triangle, isosceles, opposite, sides, angles, interior, exterior, congruent, midpoint, medians, base angles, parallel, line segment, reflections, rotations, translations, transformations	