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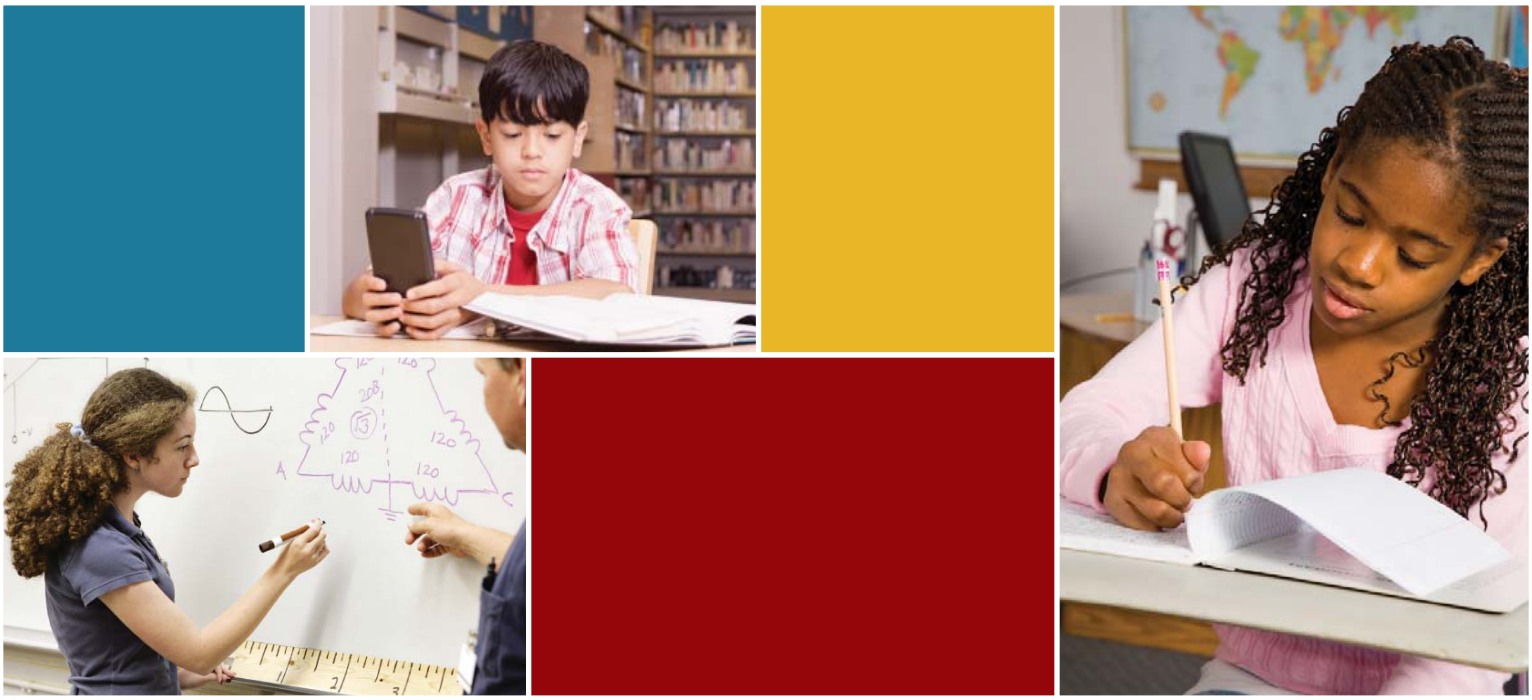
COMMON CORE STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

Colorado Academic STANDARDS

Preschool and Kindergarten

Mathematics

Mathematics



Colorado Academic Standards in Mathematics and The Common Core State Standards for Mathematics

On December 10, 2009, the Colorado State Board of Education adopted the revised Mathematics Academic Standards, along with academic standards in nine other content areas, creating Colorado's first fully aligned preschool through high school academic expectations. Developed by a broad spectrum of Coloradans representing Pre-K and K-12 education, higher education, and business, utilizing the best national and international exemplars, the intention of these standards is to prepare Colorado schoolchildren for achievement at each grade level, and ultimately, for successful performance in postsecondary institutions and/or the workforce.

Concurrent to the revision of the Colorado standards was the Common Core State Standards (CCSS) initiative, whose process and purpose significantly overlapped with that of the Colorado Academic Standards. Led by the Council of Chief State School Officers (CCSSO) and the National Governors Association (NGA), these standards present a national perspective on academic expectations for students, Kindergarten through High School in the United States.

Upon the release of the Common Core State Standards for Mathematics on June 2, 2010, the Colorado Department of Education began a gap analysis process to determine the degree to which the expectations of the Colorado Academic Standards aligned with the Common Core. The independent analysis proved a nearly 95% alignment between the two sets of standards. On August 2, 2010, the Colorado State Board of Education adopted the Common Core State Standards, and requested the integration of the Common Core State Standards and the Colorado Academic Standards.

In partnership with the dedicated members of the Colorado Standards Revision Subcommittee in Mathematics, this document represents the integration of the combined academic content of both sets of standards, maintaining the unique aspects of the Colorado Academic Standards, which include personal financial literacy, 21st century skills, school readiness competencies, postsecondary and workforce readiness competencies, and preschool expectations. The result is a world-class set of standards that are greater than the sum of their parts.

The Colorado Department of Education encourages you to review the Common Core State Standards and the extensive appendices at www.corestandards.org. While all the expectations of the Common Core State Standards are embedded and **coded with CCSS:** in this document, additional information on the development and the intentions behind the Common Core State Standards can be found on the website.

Colorado Academic Standards
Mathematics Standards

"Pure mathematics is, in its way, the poetry of logical ideas."
Albert Einstein

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"If America is to maintain our high standard of living, we must continue to innovate. We are competing with nations many times our size. We don't have a single brain to waste. Math and science are the engines of innovation. With these engines we can lead the world. We must demystify math and science so that all students feel the joy that follows understanding."
Dr. Michael Brown, Nobel Prize Laureate

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In the 21st century, a vibrant democracy depends on the full, informed participation of all people. We have a vast and rapidly growing trove of information available at any moment. However, being *informed* means, in part, using one's sense of number, shape, data and symbols to organize, interpret, make and assess the validity of claims about quantitative information. In short, informed members of society know and do mathematics.

Mathematics is indispensable for understanding our world. In addition to providing the tools of arithmetic, algebra, geometry and statistics, it offers a way of thinking about patterns and relationships of quantity and space and the connections among them. Mathematical reasoning allows us to devise and evaluate methods for solving problems, make and test conjectures about properties and relationships, and model the world around us.

Standards Organization and Construction

As the subcommittee began the revision process to improve the existing standards, it became evident that the way the standards information was organized, defined, and constructed needed to change from the existing documents. The new design is intended to provide more clarity and direction for teachers, and to show how 21st century skills and the elements of school readiness and postsecondary and workforce readiness indicators give depth and context to essential learning.

The "Continuum of State Standards Definitions" section that follows shows the hierarchical order of the standards components. The "Standards Template" section demonstrates how this continuum is put into practice.

The elements of the revised standards are:

Prepared Graduate Competencies: The preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Standard: The topical organization of an academic content area.

High School Expectations: The articulation of the concepts and skills of a standard that indicates a student is making progress toward being a prepared graduate. *What do students need to know in high school?*

Grade Level Expectations: The articulation (at each grade level), concepts, and skills of a standard that indicate a student is making progress toward being ready for high school. *What do students need to know from preschool through eighth grade?*

Evidence Outcomes: The indication that a student is meeting an expectation at the mastery level. *How do we know that a student can do it?*

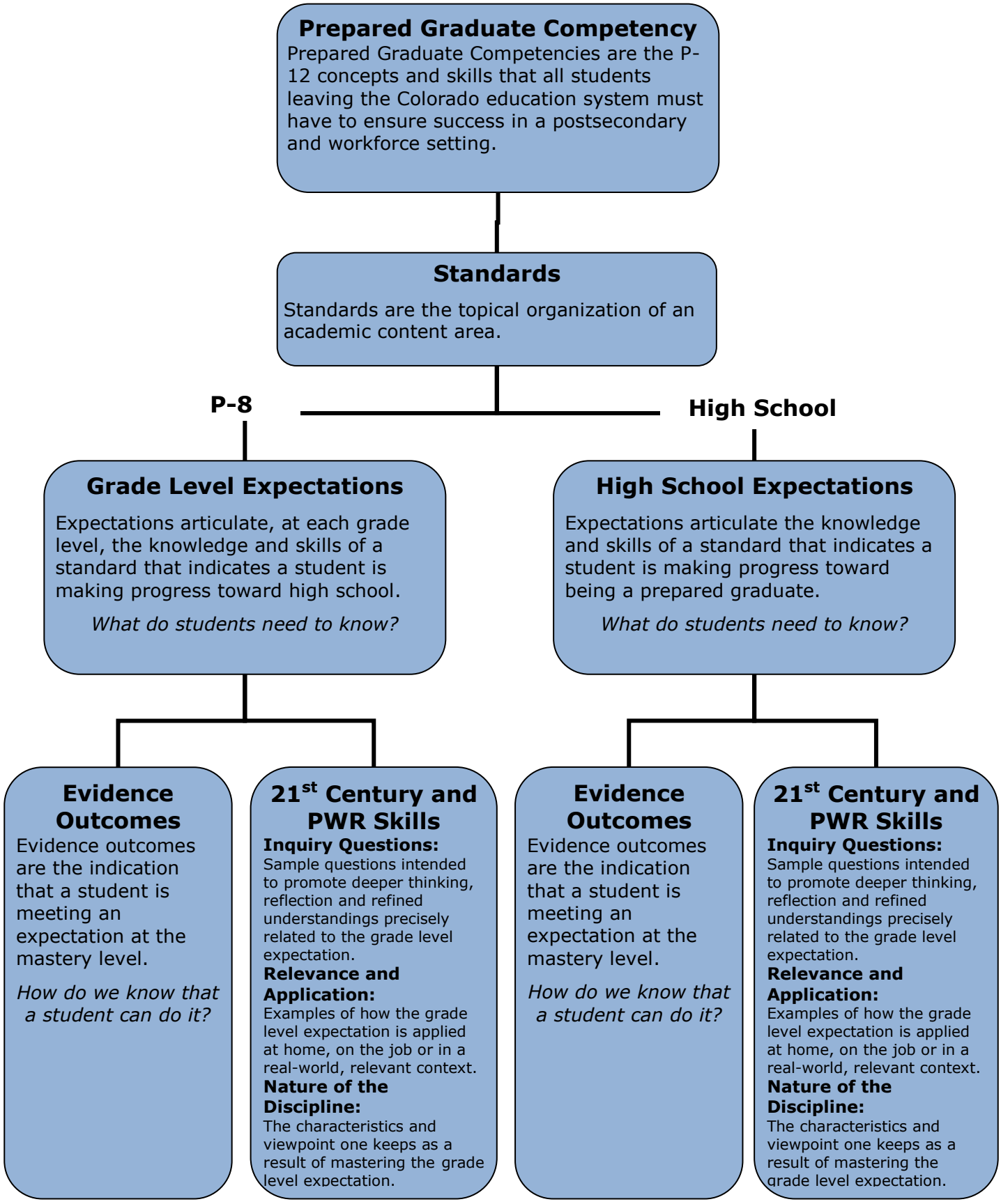
21st Century Skills and Readiness Competencies: Includes the following:

- ***Inquiry Questions:***
  Sample questions are intended to promote deeper thinking, reflection and refined understandings precisely related to the grade level expectation.

- ***Relevance and Application:***
  Examples of how the grade level expectation is applied at home, on the job or in a real-world, relevant context.

- ***Nature of the Discipline:***
  The characteristics and viewpoint one keeps as a result of mastering the grade level expectation.

Continuum of State Standards Definitions



STANDARDS TEMPLATE

Content Area: NAME OF CONTENT AREA

Standard: The topical organization of an academic content area.

Prepared Graduates:

- The P-12 concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting

High School and Grade Level Expectations

Concepts and skills students master:

Grade Level Expectation: High Schools: The articulation of the concepts and skills of a standard that indicates a student is making progress toward being a prepared graduate.

Grade Level Expectations: The articulation, at each grade level, the concepts and skills of a standard that indicates a student is making progress toward being ready for high school.

What do students need to know?

Evidence Outcomes	**21st Century Skills and Readiness Competencies**
Students can: Evidence outcomes are the indication that a student is meeting an expectation at the mastery level. *How do we know that a student can do it?*	**Inquiry Questions:** Sample questions intended to promote deeper thinking, reflection and refined understandings precisely related to the grade level expectation.
	Relevance and Application: Examples of how the grade level expectation is applied at home, on the job or in a real-world, relevant context.
	Nature of the Discipline: The characteristics and viewpoint one keeps as a result of mastering the grade level expectation.

Prepared Graduate Competencies in Mathematics

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared graduates in mathematics:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities
- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning
- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts
- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Apply transformation to numbers, shapes, functional representations, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Colorado Academic Standards
Mathematics

The Colorado academic standards in mathematics are the topical organization of the concepts and skills every Colorado student should know and be able to do throughout their preschool through twelfth-grade experience.

1. **Number Sense, Properties, and Operations**
   Number sense provides students with a firm foundation in mathematics. Students build a deep understanding of quantity, ways of representing numbers, relationships among numbers, and number systems. Students learn that numbers are governed by properties and understanding these properties leads to fluency with operations.

2. **Patterns, Functions, and Algebraic Structures**
   Pattern sense gives students a lens with which to understand trends and commonalities. Students recognize and represent mathematical relationships and analyze change. Students learn that the structures of algebra allow complex ideas to be expressed succinctly.

3. **Data Analysis, Statistics, and Probability**
   Data and probability sense provides students with tools to understand information and uncertainty. Students ask questions and gather and use data to answer them. Students use a variety of data analysis and statistics strategies to analyze, develop and evaluate inferences based on data. Probability provides the foundation for collecting, describing, and interpreting data.

4. **Shape, Dimension, and Geometric Relationships**
   Geometric sense allows students to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. Students learn that geometry and measurement are useful in representing and solving problems in the real world as well as in mathematics.

Modeling Across the Standards
Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards, specific modeling standards appear throughout the high school standards indicated by a star symbol (*).

Standards for Mathematical Practice
from
The Common Core State Standards for Mathematics

The Standards for Mathematical Practice have been included in the Nature of Mathematics section in each Grade Level Expectation of the Colorado Academic Standards. The following definitions and explanation of the Standards for Mathematical Practice from the Common Core State Standards can be found on pages 6, 7, and 8 in the Common Core State Standards for Mathematics. Each Mathematical Practices statement has been notated with (MP) at the end of the statement.

Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose.

Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions,
explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression x^2 + 9x + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or

as being composed of several objects. For example, they can see 5 – 3(x – y)^2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y – 2)/(x – 1) = 3. Noticing the regularity in the way terms cancel when expanding (x – 1)(x + 1), (x – 1)(x^2 + x + 1), and (x – 1)(x^3 + x^2 + x + 1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content
The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction. The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Mathematics
Grade Level Expectations at a Glance

Standard	Grade Level Expectation
Kindergarten	
1. Number Sense, Properties, and Operations	1. Whole numbers can be used to name, count, represent, and order quantity 2. Composing and decomposing quantity forms the foundation for addition and subtraction
2. Patterns, Functions, and Algebraic Structures	Expectations for this standard are integrated into the other standards at this grade level.
3. Data Analysis, Statistics, and Probability	Expectations for this standard are integrated into the other standards at this grade level.
4. Shape, Dimension, and Geometric Relationships	1. Shapes are described by their characteristics and position and created by composing and decomposing 2. Measurement is used to compare and order objects

From the Common State Standards for Mathematics, Page 9.

Mathematics | Kindergarten
In Kindergarten, instructional time should focus on two critical areas: (1) representing, relating, and operating on whole numbers, initially with sets of objects; (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics.

(1) Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as 5 + 2 = 7 and 7 – 2 = 5. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

(2) Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

Mathematics
Grade Level Expectations at a Glance

Standard	Grade Level Expectation
Preschool	
1. Number Sense, Properties, and Operations	1. Quantities can be represented and counted
2. Patterns, Functions, and Algebraic Structures	Expectations for this standard are integrated into the other standards at this grade level.
3. Data Analysis, Statistics, and Probability	Expectations for this standard are integrated into the other standards at this grade level.
4. Shape, Dimension, and Geometric Relationships	1. Shapes can be observed in the world and described in relation to one another 2. Measurement is used to compare objects

21st Century Skills and Readiness Competencies in Mathematics

Mathematics in Colorado's description of 21st century skills is a synthesis of the essential abilities students must apply in our rapidly changing world. Today's mathematics students need a repertoire of knowledge and skills that are more diverse, complex, and integrated than any previous generation. Mathematics is inherently demonstrated in each of Colorado 21st century skills, as follows:

Critical Thinking and Reasoning
Mathematics is a discipline grounded in critical thinking and reasoning. Doing mathematics involves recognizing problematic aspects of situations, devising and carrying out strategies, evaluating the reasonableness of solutions, and justifying methods, strategies, and solutions. Mathematics provides the grammar and structure that make it possible to describe patterns that exist in nature and society.

Information Literacy
The discipline of mathematics equips students with tools and habits of mind to organize and interpret quantitative data. Informationally literate mathematics students effectively use learning tools, including technology, and clearly communicate using mathematical language.

Collaboration
Mathematics is a social discipline involving the exchange of ideas. In the course of doing mathematics, students offer ideas, strategies, solutions, justifications, and proofs for others to evaluate. In turn, the mathematics student interprets and evaluates the ideas, strategies, solutions, justifications and proofs of others.

Self-Direction
Doing mathematics requires a productive disposition and self-direction. It involves monitoring and assessing one's mathematical thinking and persistence in searching for patterns, relationships, and sensible solutions.

Invention
Mathematics is a dynamic discipline, ever expanding as new ideas are contributed. Invention is the key element as students make and test conjectures, create mathematical models of real-world phenomena, generalize results, and make connections among ideas, strategies and solutions.

Colorado's Description for School Readiness
(Adopted by the State Board of Education, December 2008)
School readiness describes both the preparedness of a child to engage in and benefit from learning experiences, and the ability of a school to meet the needs of all students enrolled in publicly funded preschools or kindergartens. School readiness is enhanced when schools, families, and community service providers work collaboratively to ensure that every child is ready for higher levels of learning in academic content.

Colorado's Description of Postsecondary and Workforce Readiness
(Adopted by the State Board of Education, June 2009)
Postsecondary and workforce readiness describes the knowledge, skills, and behaviors essential for high school graduates to be prepared to enter college and the workforce and to compete in the global economy. The description assumes students have developed consistent intellectual growth throughout their high school career as a result of academic work that is increasingly challenging, engaging, and coherent. Postsecondary education and workforce readiness assumes that students are ready and able to demonstrate the following without the need for remediation: Critical thinking and problem-solving; finding and using information/information technology; creativity and innovation; global and cultural awareness; civic responsibility; work ethic; personal responsibility; communication; and collaboration.

How These Skills and Competencies are Embedded in the Revised Standards
Three themes are used to describe these important skills and competencies and are interwoven throughout the standards: *inquiry questions; relevance and application; and the nature of each discipline.* These competencies should not be thought of stand-alone concepts, but should be integrated throughout the curriculum in all grade levels. Just as it is impossible to teach thinking skills to students without the content to think about, it is equally impossible for students to understand the content of a discipline without grappling with complex questions and the investigation of topics.

Inquiry Questions – Inquiry is a multifaceted process requiring students to think and pursue understanding. Inquiry demands that students (a) engage in an active observation and questioning process; (b) investigate to gather evidence; (c) formulate explanations based on evidence; (d) communicate and justify explanations, and; (e) reflect and refine ideas. Inquiry is more than hands-on activities; it requires students to cognitively wrestle with core concepts as they make sense of new ideas.

Relevance and Application – The hallmark of learning a discipline is the ability to apply the knowledge, skills, and concepts in real-world, relevant contexts. Components of this include solving problems, developing, adapting, and refining solutions for the betterment of society. The application of a discipline, including how technology assists or accelerates the work, enables students to more fully appreciate how the mastery of the grade level expectation matters after formal schooling is complete.

Nature of Discipline – The unique advantage of a discipline is the perspective it gives the mind to see the world and situations differently. The characteristics and viewpoint one keeps as a result of mastering the grade level expectation is the nature of the discipline retained in the mind's eye.

1.Number Sense, Properties, and Operations

Number sense provides students with a firm foundation in mathematics. Students build a deep understanding of quantity, ways of representing numbers, relationships among numbers, and number systems. Students learn that numbers are governed by properties, and understanding these properties leads to fluency with operations.

Prepared Graduates
The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the Number Sense, Properties, and Operations Standard are:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities
- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Apply transformation to numbers, shapes, functional representations, and data

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Kindergarten

Concepts and skills students master:

1. Whole numbers can be used to name, count, represent, and order quantity

Evidence Outcomes	**21st Century Skills and Readiness Competencies**
Students can: a. Use number names and the count sequence. (CCSS: K.CC) i. Count to 100 by ones and by tens. (CCSS: K.CC.1) ii. Count forward beginning from a given number within the known sequence.[1] (CCSS: K.CC.2) iii. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20.[2] (CCSS: K.CC.3) b. Count to determine the number of objects. (CCSS: K.CC) i. Apply the relationship between numbers and quantities and connect counting to cardinality.[3] (CCSS: K.CC.4) ii. Count and represent objects to 20.[4] (CCSS: K.CC.5) c. Compare and instantly recognize numbers. (CCSS: K.CC) i. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group.[5] (CCSS: K.CC.6) ii. Compare two numbers between 1 and 10 presented as written numerals. (CCSS: K.CC.7) iii. Identify small groups of objects fewer than five without counting	**Inquiry Questions:** 1. Why do we count things? 2. Is there a wrong way to count? Why? 3. How do you know when you have more or less? 4. What does it mean to be second and how is it different than two? **Relevance and Application:** 1. Counting is used constantly in everyday life such as counting plates for the dinner table, people on a team, pets in the home, or trees in a yard. 2. Numerals are used to represent quantities. 3. People use numbers to communicate with others such as two more forks for the dinner table, one less sister than my friend, or six more dollars for a new toy. **Nature of Mathematics:** 1. Mathematics involves visualization and representation of ideas. 2. Numbers are used to count and order both real and imaginary objects. 3. Mathematicians attend to precision. (MP) 4. Mathematicians look for and make use of structure. (MP)

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Apply transformation to numbers, shapes, functional representations, and data

Grade Level Expectation: Kindergarten

Concepts and skills students master:

2. Composing and decomposing quantity forms the foundation for addition and subtraction

Evidence Outcomes	**21st Century Skills and Readiness Competencies**
Students can: a. Model and describe addition as putting together and adding to, and subtraction as taking apart and taking from, using objects or drawings. (CCSS: K.OA) i. Represent addition and subtraction with objects, fingers, mental images, drawings, sounds,[6] acting out situations, verbal explanations, expressions, or equations. (CCSS: K.OA.1) ii. Solve addition and subtraction word problems, and add and subtract within 10.[7] (CCSS: K.OA.2) iii. Decompose numbers less than or equal to 10 into pairs in more than one way.[8] (CCSS: K.OA.3) iv. For any number from 1 to 9, find the number that makes 10 when added to the given number.[9] (CCSS: K.OA.4) v. Use objects including coins and drawings to model addition and subtraction problems to 10 (PFL) b. Fluently add and subtract within 5. (CCSS: K.OA.5) c. Compose and decompose numbers 11–19 to gain foundations for place value using objects and drawings.[10] (CCSS: K.NBT)	**Inquiry Questions:** 1. What happens when two quantities are combined? 2. What happens when a set of objects is separated into different sets? **Relevance and Application:** 1. People combine quantities to find a total such as number of boys and girls in a classroom or coins for a purchase. 2. People use subtraction to find what is left over such as coins left after a purchase, number of toys left after giving some away. **Nature of Mathematics:** 1. Mathematicians create models of problems that reveal relationships and meaning. 2. Mathematics involves the creative use of imagination. 3. Mathematicians reason abstractly and quantitatively. (MP) 4. Mathematicians model with mathematics. (MP)

Standard: 1. Number Sense, Properties, and Operations
Kindergarten

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[1] instead of having to begin at 1. (CCSS: K.CC.2)
[2] with 0 representing a count of no objects. (CCSS: K.CC.3)
[3] When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. (CCSS: K.CC.4a)
Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. (CCSS: K.CC.4b)
Understand that each successive number name refers to a quantity that is one larger. (CCSS: K.CC.4c)
[4] Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration. (CCSS: K.CC.5)
Given a number from 1–20, count out that many objects. (CCSS: K.CC.5)
[5] e.g., by using matching and counting strategies. (CCSS: K.CC.6)
[6] e.g., claps. (CCSS: K.OA.1)
[7] e.g., by using objects or drawings to represent the problem. (CCSS: K.OA.2)
[8] e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1). (CCSS: K.OA.3)
[9] e.g., by using objects or drawings, and record the answer with a drawing or equation. (CCSS: K.OA.4)
[10] Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones. (CCSS: K.NBT.1)

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Preschool

Concepts and skills students master:

1. Quantities can be represented and counted

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Count and represent objects including coins to 10 (PFL) b. Match a quantity with a numeral	**Inquiry Questions:** 1. What do numbers tell us? 2. Is there a biggest number?
	Relevance and Application: 1. Counting helps people to determine how many such as how big a family is, how many pets there are, such as how many members in one's family, how many mice on the picture book page, how many counting bears in the cup. 2. People sort things to make sense of sets of things such as sorting pencils, toys, or clothes.
	Nature of Mathematics: 1. Numbers are used to count and order objects. 2. Mathematicians reason abstractly and quantitatively. (MP) 3. Mathematicians attend to precision. (MP)

2. Patterns, Functions, and Algebraic Structures

Pattern sense gives students a lens with which to understand trends and commonalities. Being a student of mathematics involves recognizing and representing mathematical relationships and analyzing change. Students learn that the structures of algebra allow complex ideas to be expressed succinctly.

Prepared Graduates
The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must have to ensure success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the 2. Patterns, Functions, and Algebraic Structures Standard are:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

Grade Level Expectation: PRESCHOOL THROUGH THIRD GRADE

Concepts and skills students master:

Evidence Outcomes	**21st Century Skills and Readiness Competencies**
Students can: **Expectations for this standard are integrated into the other standards at preschool through third grade.**	**Inquiry Questions:**
	Relevance and Application:
	Nature of Mathematics:

3. Data Analysis, Statistics, and Probability

Data and probability sense provides students with tools to understand information and uncertainty. Students ask questions and gather and use data to answer them. Students use a variety of data analysis and statistics strategies to analyze, develop and evaluate inferences based on data. Probability provides the foundation for collecting, describing, and interpreting data.

Prepared Graduates
The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the 3. Data Analysis, Statistics, and Probability Standard are:

- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts
- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data
- Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

Grade Level Expectation: PRESCHOOL AND KINDERGARTEN

Concepts and skills students master:

Evidence Outcomes	**21st Century Skills and Readiness Competencies**
Students can: **Expectations for this standard are integrated into the other standards at preschool through kindergarten.**	**Inquiry Questions:**
	Relevance and Application:
	Nature of Mathematics:

4. Shape, Dimension, and Geometric Relationships

Geometric sense allows students to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. Students learn that geometry and measurement are useful in representing and solving problems in the real world as well as in mathematics.

Prepared Graduates
The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the 4. Shape, Dimension, and Geometric Relationships standard are:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Apply transformation to numbers, shapes, functional representations, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: Kindergarten

Concepts and skills students master:

1. Shapes can be described by characteristics and position and created by composing and decomposing

Evidence Outcomes	**21st Century Skills and Readiness Competencies**
Students can: a. Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres). (CCSS: K.G) i. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*. (CCSS: K.G.1) ii. Correctly name shapes regardless of their orientations or overall size. (CCSS: K.G.2) iii. Identify shapes as two-dimensional[1] or three dimensional.[2] (CCSS: K.G.3) b. Analyze, compare, create, and compose shapes. (CCSS: K.G) i. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts[3] and other attributes.[4] (CCSS: K.G.4) ii. Model shapes in the world by building shapes from components[5] and drawing shapes. (CCSS: K.G.5) iii. Compose simple shapes to form larger shapes.[6] (CCSS: K.G.6)	**Inquiry Questions:** 1. What are the ways to describe where an object is? 2. What are all the things you can think of that are round? What is the same about these things? 3. How are these shapes alike and how are they different? 4. Can you make one shape with other shapes? **Relevance and Application:** 1. Shapes help people describe the world. For example, a box is a cube, the Sun looks like a circle, and the side of a dresser looks like a rectangle. 2. People communicate where things are by their location in space using words like next to, below, or between. **Nature of Mathematics:** 1. Geometry helps discriminate one characteristic from another. 2. Geometry clarifies relationships between and among different objects. 3. Mathematicians model with mathematics. (MP) 4. Mathematicians look for and make use of structure. (MP)

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Kindergarten

Concepts and skills students master:

2. Measurement is used to compare and order objects

Evidence Outcomes	**21st Century Skills and Readiness Competencies**
Students can: a. Describe and compare measurable attributes. (CCSS: K.MD) i. Describe measurable attributes of objects, such as length or weight. (CCSS: K.MD.1) ii. Describe several measurable attributes of a single object. (CCSS: K.MD.1) iii. Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference.[7] (CCSS: K.MD.2) iv. Order several objects by length, height, weight, or price (PFL) b. Classify objects and count the number of objects in each category. (CCSS: K.MD) i. Classify objects into given categories. (CCSS: K.MD.3) ii. Count the numbers of objects in each category. (CCSS: K.MD.3) iii. Sort the categories by count. (CCSS: K.MD.3)	**Inquiry Questions:** 1. How can you tell when one thing is bigger than another? 2. How is height different from length? **Relevance and Application:** 1. Measurement helps to understand and describe the world such as in cooking, playing, or pretending. 2. People compare objects to communicate and collaborate with others. For example, we describe items like the long ski, the heavy book, the expensive toy. **Nature of Mathematics:** 1. A system of measurement provides a common language that everyone can use to communicate about objects. 2. Mathematicians use appropriate tools strategically. (MP) 3. Mathematicians attend to precision. (MP)

Standard: 4. Shape, Dimension, and Geometric Relationships
Kindergarten

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[1] lying in a plane, "flat". (CCSS: K.G.3)
[2] "solid". (CCSS: K.G.3)
[3] e.g., number of sides and vertices/"corners". (CCSS: K.G.4)
[4] e.g., having sides of equal length. (CCSS: K.G.4)
[5] e.g., sticks and clay balls. (CCSS: K.G.5)
[6] For example, "Can you join these two triangles with full sides touching to make a rectangle?" (CCSS: K.G.6)
[7] For example, directly compare the heights of two children and describe one child as taller/shorter. (CCSS: K.MD.2)

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data

Grade Level Expectation: Preschool

Concepts and skills students master:

1. Shapes can be observed in the world and described in relation to one another

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Match, sort, group and name basic shapes found in the natural environment b. Sort similar groups of objects into simple categories based on attributes c. Use words to describe attributes of objects d. Follow directions to arrange, order, or position objects	**Inquiry Questions:** 1. How do we describe where something is? 2. Where do you see shapes around you? 3. How can we arrange these shapes? 4. Why do we put things in a group? 5. What is the same about these objects and what is different? 6. What are the ways to sort objects?
	Relevance and Application: 1. Shapes and position help students describe and understand the environment such as in cleaning up, or organizing and arranging their space. 2. Comprehension of order and position helps students learn to follow directions. 3. Technology games can be used to arrange and position objects. 4. Sorting and grouping allows people to organize their world. For example, we set up time for clean up, and play.
	Nature of Mathematics: 1. Geometry affords the predisposition to explore and experiment. 2. Mathematicians organize objects in different ways to learn about the objects and a group of objects. 3. Mathematicians attend to precision. (MP) 4. Mathematicians look for and make use of structure. (MP)

Content Area: Mathematics

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Preschool

Concepts and skills students master:

2. Measurement is used to compare objects

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Describe the order of common events b. Group objects according to their size using standard and non-standard forms (height, weight, length, or color brightness) of measurement c. Sort coins by physical attributes such as color or size (PFL)	**Inquiry Questions:** 1. How do we know how big something is? 2. How do we describe when things happened? **Applying Mathematics in Society and Using Technology:** 1. Understanding the order of events allows people to tell a story or communicate about the events of the day. 2. Measurements helps people communicate about the world. For example, we describe items like big and small cars, short and long lines, or heavy and light boxes. **Nature of Mathematics:** 1. Mathematicians sort and organize to create patterns. Mathematicians look for patterns and regularity. The search for patterns can produce rewarding shortcuts and mathematical insights. 2. Mathematicians reason abstractly and quantitatively. (MP) 3. Mathematicians use appropriate tools strategically. (MP)

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