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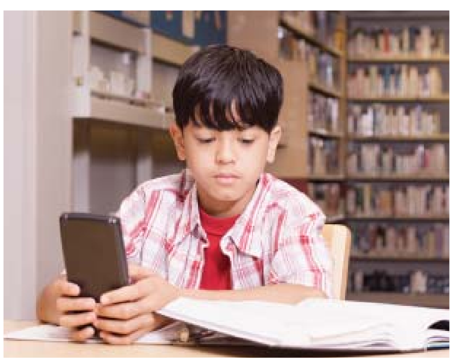
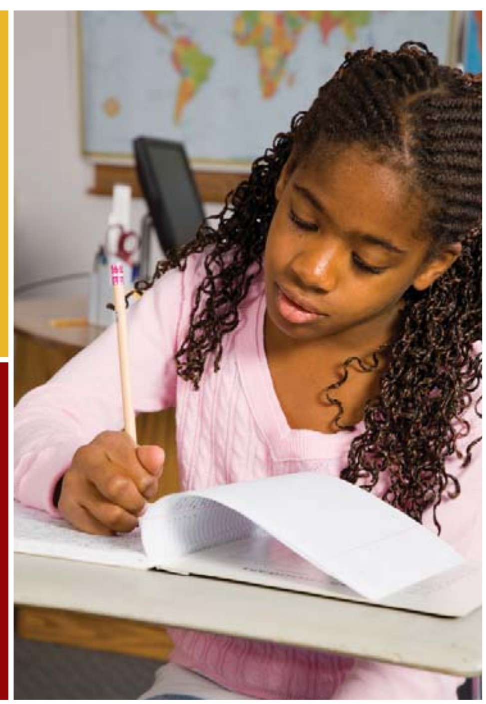
COMMON CORE STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

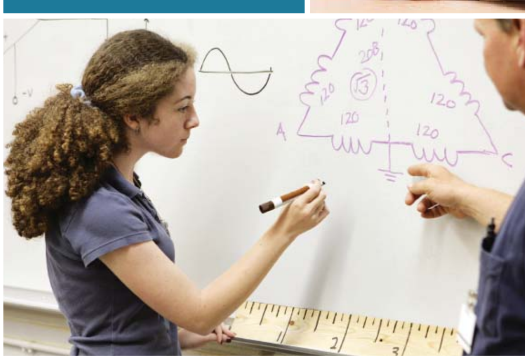
Colorado Academic STANDARDS

Eighth Grade

Mathematics







Colorado Academic Standards in Mathematics
and
The Common Core State Standards for Mathematics

On December 10, 2009, the Colorado State Board of Education adopted the revised Mathematics Academic Standards, along with academic standards in nine other content areas, creating Colorado's first fully aligned preschool through high school academic expectations. Developed by a broad spectrum of Coloradans representing Pre-K and K-12 education, higher education, and business, utilizing the best national and international exemplars, the intention of these standards is to prepare Colorado schoolchildren for achievement at each grade level, and ultimately, for successful performance in postsecondary institutions and/or the workforce.

Concurrent to the revision of the Colorado standards was the Common Core State Standards (CCSS) initiative, whose process and purpose significantly overlapped with that of the Colorado Academic Standards. Led by the Council of Chief State School Officers (CCSSO) and the National Governors Association (NGA), these standards present a national perspective on academic expectations for students, Kindergarten through High School in the United States.

Upon the release of the Common Core State Standards for Mathematics on June 2, 2010, the Colorado Department of Education began a gap analysis process to determine the degree to which the expectations of the Colorado Academic Standards aligned with the Common Core. The independent analysis proved a nearly 95% alignment between the two sets of standards. On August 2, 2010, the Colorado State Board of Education adopted the Common Core State Standards, and requested the integration of the Common Core State Standards and the Colorado Academic Standards.

In partnership with the dedicated members of the Colorado Standards Revision Subcommittee in Mathematics, this document represents the integration of the combined academic content of both sets of standards, maintaining the unique aspects of the Colorado Academic Standards, which include personal financial literacy, 21st century skills, school readiness competencies, postsecondary and workforce readiness competencies, and preschool expectations. The result is a world-class set of standards that are greater than the sum of their parts.

The Colorado Department of Education encourages you to review the Common Core State Standards and the extensive appendices at www.corestandards.org. While all the expectations of the Common Core State Standards are embedded and **coded with CCSS:** in this document, additional information on the development and the intentions behind the Common Core State Standards can be found on the website.

Colorado Academic Standards
Mathematics Standards

"Pure mathematics is, in its way, the poetry of logical ideas."
Albert Einstein

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"If America is to maintain our high standard of living, we must continue to innovate. We are competing with nations many times our size. We don't have a single brain to waste. Math and science are the engines of innovation. With these engines we can lead the world. We must demystify math and science so that all students feel the joy that follows understanding."
Dr. Michael Brown, Nobel Prize Laureate

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In the 21st century, a vibrant democracy depends on the full, informed participation of all people. We have a vast and rapidly growing trove of information available at any moment. However, being *informed* means, in part, using one's sense of number, shape, data and symbols to organize, interpret, make and assess the validity of claims about quantitative information. In short, informed members of society know and do mathematics.

Mathematics is indispensable for understanding our world. In addition to providing the tools of arithmetic, algebra, geometry and statistics, it offers a way of thinking about patterns and relationships of quantity and space and the connections among them. Mathematical reasoning allows us to devise and evaluate methods for solving problems, make and test conjectures about properties and relationships, and model the world around us.

Standards Organization and Construction

As the subcommittee began the revision process to improve the existing standards, it became evident that the way the standards information was organized, defined, and constructed needed to change from the existing documents. The new design is intended to provide more clarity and direction for teachers, and to show how 21st century skills and the elements of school readiness and postsecondary and workforce readiness indicators give depth and context to essential learning.

The "Continuum of State Standards Definitions" section that follows shows the hierarchical order of the standards components. The "Standards Template" section demonstrates how this continuum is put into practice.

The elements of the revised standards are:

Prepared Graduate Competencies: The preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Standard: The topical organization of an academic content area.

High School Expectations: The articulation of the concepts and skills of a standard that indicates a student is making progress toward being a prepared graduate. *What do students need to know in high school?*

Grade Level Expectations: The articulation (at each grade level), concepts, and skills of a standard that indicate a student is making progress toward being ready for high school. *What do students need to know from preschool through eighth grade?*

Evidence Outcomes: The indication that a student is meeting an expectation at the mastery level. *How do we know that a student can do it?*

21st Century Skills and Readiness Competencies: Includes the following:

- ***Inquiry Questions:***
  Sample questions are intended to promote deeper thinking, reflection and refined understandings precisely related to the grade level expectation.

- ***Relevance and Application:***
  Examples of how the grade level expectation is applied at home, on the job or in a real-world, relevant context.

- ***Nature of the Discipline:***
  The characteristics and viewpoint one keeps as a result of mastering the grade level expectation.

Continuum of State Standards Definitions

Prepared Graduate Competency
Prepared Graduate Competencies are the P-12 concepts and skills that all students leaving the Colorado education system must have to ensure success in a postsecondary and workforce setting.

Standards
Standards are the topical organization of an academic content area.

P-8

Grade Level Expectations
Expectations articulate, at each grade level, the knowledge and skills of a standard that indicates a student is making progress toward high school.

What do students need to know?

Evidence Outcomes
Evidence outcomes are the indication that a student is meeting an expectation at the mastery level.

How do we know that a student can do it?

21st Century and PWR Skills
Inquiry Questions:
Sample questions intended to promote deeper thinking, reflection and refined understandings precisely related to the grade level expectation.

Relevance and Application:
Examples of how the grade level expectation is applied at home, on the job or in a real-world, relevant context.

Nature of the Discipline:
The characteristics and viewpoint one keeps as a result of mastering the grade level expectation.

High School

High School Expectations
Expectations articulate the knowledge and skills of a standard that indicates a student is making progress toward being a prepared graduate.

What do students need to know?

Evidence Outcomes
Evidence outcomes are the indication that a student is meeting an expectation at the mastery level.

How do we know that a student can do it?

21st Century and PWR Skills
Inquiry Questions:
Sample questions intended to promote deeper thinking, reflection and refined understandings precisely related to the grade level expectation.

Relevance and Application:
Examples of how the grade level expectation is applied at home, on the job or in a real-world, relevant context.

Nature of the Discipline:
The characteristics and viewpoint one keeps as a result of mastering the grade level expectation.

STANDARDS TEMPLATE

Content Area: NAME OF CONTENT AREA
Standard: The topical organization of an academic content area.

Prepared Graduates:

- ➢ The P-12 concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting

High School and Grade Level Expectations

Concepts and skills students master:

Grade Level Expectation: High Schools: The articulation of the concepts and skills of a standard that indicates a student is making progress toward being a prepared graduate.

Grade Level Expectations: The articulation, at each grade level, the concepts and skills of a standard that indicates a student is making progress toward being ready for high school.

What do students need to know?

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: Evidence outcomes are the indication that a student is meeting an expectation at the mastery level. *How do we know that a student can do it?*	**Inquiry Questions:** Sample questions intended to promote deeper thinking, reflection and refined understandings precisely related to the grade level expectation.
	Relevance and Application: Examples of how the grade level expectation is applied at home, on the job or in a real-world, relevant context.
	Nature of the Discipline: The characteristics and viewpoint one keeps as a result of mastering the grade level expectation.

Prepared Graduate Competencies in Mathematics

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared graduates in mathematics:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities
- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning
- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts
- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Apply transformation to numbers, shapes, functional representations, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Colorado Academic Standards
Mathematics

The Colorado academic standards in mathematics are the topical organization of the concepts and skills every Colorado student should know and be able to do throughout their preschool through twelfth-grade experience.

1. **Number Sense, Properties, and Operations**
   Number sense provides students with a firm foundation in mathematics. Students build a deep understanding of quantity, ways of representing numbers, relationships among numbers, and number systems. Students learn that numbers are governed by properties and understanding these properties leads to fluency with operations.

2. **Patterns, Functions, and Algebraic Structures**
   Pattern sense gives students a lens with which to understand trends and commonalities. Students recognize and represent mathematical relationships and analyze change. Students learn that the structures of algebra allow complex ideas to be expressed succinctly.

3. **Data Analysis, Statistics, and Probability**
   Data and probability sense provides students with tools to understand information and uncertainty. Students ask questions and gather and use data to answer them. Students use a variety of data analysis and statistics strategies to analyze, develop and evaluate inferences based on data. Probability provides the foundation for collecting, describing, and interpreting data.

4. **Shape, Dimension, and Geometric Relationships**
   Geometric sense allows students to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. Students learn that geometry and measurement are useful in representing and solving problems in the real world as well as in mathematics.

Modeling Across the Standards
Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards, specific modeling standards appear throughout the high school standards indicated by a star symbol (*).

Standards for Mathematical Practice
from
The Common Core State Standards for Mathematics

The Standards for Mathematical Practice have been included in the Nature of Mathematics section in each Grade Level Expectation of the Colorado Academic Standards. The following definitions and explanation of the Standards for Mathematical Practice from the Common Core State Standards can be found on pages 6, 7, and 8 in the Common Core State Standards for Mathematics. Each Mathematical Practices statement has been notated with (MP) at the end of the statement.

Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose.

Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 *equals the well remembered* $7 \times 5 + 7 \times 3$*, in preparation for learning about the distributive property. In the expression* $x^2 + 9x + 14$*, older students can see the 14 as* 2×7 *and the 9 as* $2 + 7$*. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or*

as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction. The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Mathematics
Grade Level Expectations at a Glance

Standard	Grade Level Expectation
Eighth Grade	
1. Number Sense, Properties, and Operations	1. In the real number system, rational and irrational numbers are in one to one correspondence to points on the number line
2. Patterns, Functions, and Algebraic Structures	1. Linear functions model situations with a constant rate of change and can be represented numerically, algebraically, and graphically 2. Properties of algebra and equality are used to solve linear equations and systems of equations 3. Graphs, tables and equations can be used to distinguish between linear and nonlinear functions
3. Data Analysis, Statistics, and Probability	1. Visual displays and summary statistics of two-variable data condense the information in data sets into usable knowledge
4. Shape, Dimension, and Geometric Relationships	1. Transformations of objects can be used to define the concepts of congruence and similarity 2. Direct and indirect measurement can be used to describe and make comparisons

From the Common State Standards for Mathematics, Page 52.

Mathematics | Grade 8

In Grade 8, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

(1) Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions (y/x = m or y = mx) as special linear equations (y = mx + b), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount m·A. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation. Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

(2) Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and

graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

(3) Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

1.Number Sense, Properties, and Operations

Number sense provides students with a firm foundation in mathematics. Students build a deep understanding of quantity, ways of representing numbers, relationships among numbers, and number systems. Students learn that numbers are governed by properties, and understanding these properties leads to fluency with operations.

Prepared Graduates
The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the Number Sense, Properties, and Operations Standard are:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities
- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Apply transformation to numbers, shapes, functional representations, and data

Content Area: Mathematics
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Eighth Grade

Concepts and skills students master:

1. In the real number system, rational and irrational numbers are in one to one correspondence to points on the number line

Evidence Outcomes	21st Century Skills and Readiness Competencies

Students can:
a. Define irrational numbers.[1]
b. Demonstrate informally that every number has a decimal expansion. (CCSS: 8.NS.1)
  i. For rational numbers show that the decimal expansion repeats eventually. (CCSS: 8.NS.1)
  ii. Convert a decimal expansion which repeats eventually into a rational number. (CCSS: 8.NS.1)
c. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions.[2] (CCSS: 8.NS.2)
d. Apply the properties of integer exponents to generate equivalent numerical expressions.[3] (CCSS: 8.EE.1)
e. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. (CCSS: 8.EE.2)
f. Evaluate square roots of small perfect squares and cube roots of small perfect cubes.[4] (CCSS: 8.EE.2)
g. Use numbers expressed in the form of a single digit times a whole-number power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.[5] (CCSS: 8.EE.3)
h. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. (CCSS: 8.EE.4)
  i. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities.[6] (CCSS: 8.EE.4)
  ii. Interpret scientific notation that has been generated by technology. (CCSS: 8.EE.4)
	Inquiry Questions:
	1. Why are real numbers represented by a number line and why are the integers represented by points on the number line?
	2. Why is there no real number closest to zero?
	3. What is the difference between rational and irrational numbers?
	
	Relevance and Application:
	1. Irrational numbers have applications in geometry such as the length of a diagonal of a one by one square, the height of an equilateral triangle, or the area of a circle.
	2. Different representations of real numbers are used in contexts such as measurement (metric and customary units), business (profits, network down time, productivity), and community (voting rates, population density).
	3. Technologies such as calculators and computers enable people to order and convert easily among fractions, decimals, and percents.
	
	Nature of Mathematics:
	1. Mathematics provides a precise language to describe objects and events and the relationships among them.
	2. Mathematicians reason abstractly and quantitatively. (MP)
	3. Mathematicians use appropriate tools strategically. (MP)
	4. Mathematicians attend to precision. (MP)

Standard: 1. Number Sense, Properties, and Operations
Eighth Grade

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[1] Know that numbers that are not rational are called irrational. (CCSS: 8.NS.1)
[2] e.g., π^2. (CCSS: 8.NS.2)
For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations. (CCSS: 8.NS.2)
[3] For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$. (CCSS: 8.EE.1)
[4] Know that √2 is irrational. (CCSS: 8.EE.2)
[5] For example, estimate the population of the United States as 3 times 10^8 and the population of the world as 7 times 10^9, and determine that the world population is more than 20 times larger. *(CCSS: 8.EE.3)*
[6] e.g., use millimeters per year for seafloor spreading. (CCSS: 8.EE.4)

2. Patterns, Functions, and Algebraic Structures

Pattern sense gives students a lens with which to understand trends and commonalities. Being a student of mathematics involves recognizing and representing mathematical relationships and analyzing change. Students learn that the structures of algebra allow complex ideas to be expressed succinctly.

Prepared Graduates
The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must have to ensure success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the 2. Patterns, Functions, and Algebraic Structures Standard are:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Content Area: Mathematics
Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations

Grade Level Expectation: Eighth Grade

Concepts and skills students master:

1. Linear functions model situations with a constant rate of change and can be represented numerically, algebraically, and graphically

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Describe the connections between proportional relationships, lines, and linear equations. (CCSS: 8.EE) b. Graph proportional relationships, interpreting the unit rate as the slope of the graph. (CCSS: 8.EE.5) c. Compare two different proportional relationships represented in different ways.[1] (CCSS: 8.EE.5) d. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane. (CCSS: 8.EE.6) e. Derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b. (CCSS: 8.EE.6)	**Inquiry Questions:** 1. How can different representations of linear patterns present different perspectives of situations? 2. How can a relationship be analyzed with tables, graphs, and equations? 3. Why is one variable dependent upon the other in relationships? **Relevance and Application:** 1. Fluency with different representations of linear patterns allows comparison and contrast of linear situations such as service billing rates from competing companies or simple interest on savings or credit. 2. Understanding slope as rate of change allows individuals to develop and use a line of best fit for data that appears to be linearly related. 3. The ability to recognize slope and y-intercept of a linear function facilitates graphing the function or writing an equation that describes the function. **Nature of Mathematics:** 1. Mathematicians represent functions in multiple ways to gain insights into the relationships they model. 2. Mathematicians model with mathematics. (MP)

Content Area: Mathematics
Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

Grade Level Expectation: Eighth Grade

Concepts and skills students master:

2. Properties of algebra and equality are used to solve linear equations and systems of equations

Evidence Outcomes	21[st] Century Skills and Readiness Competencies
Students can: a. Solve linear equations in one variable. (CCSS: 8.EE.7) i. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions.[2] (CCSS: 8.EE.7a) ii. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. (CCSS: 8.EE.7b) b. Analyze and solve pairs of simultaneous linear equations. (CCSS: 8.EE.8) i. Explain that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. (CCSS: 8.EE.8a) ii. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.[3] (CCSS: 8.EE.8b) iii. Solve real-world and mathematical problems leading to two linear equations in two variables.[4] (CCSS: 8.EE.8c)	**Inquiry Questions:** 1. What makes a solution strategy both efficient and effective? 2. How is it determined if multiple solutions to an equation are valid? 3. How does the context of the problem affect the reasonableness of a solution? 4. Why can two equations be added together to get another true equation? **Relevance and Application:** 1. The understanding and use of equations, inequalities, and systems of equations allows for situational analysis and decision-making. For example, it helps people choose cell phone plans, calculate credit card interest and payments, and determine health insurance costs. 2. Recognition of the significance of the point of intersection for two linear equations helps to solve problems involving two linear rates such as determining when two vehicles traveling at constant speeds will be in the same place, when two calling plans cost the same, or the point when profits begin to exceed costs. **Nature of Mathematics:** 1. Mathematics involves visualization. 2. Mathematicians use tools to create visual representations of problems and ideas that reveal relationships and meaning. 3. Mathematicians make sense of problems and persevere in solving them. (MP) 4. Mathematicians use appropriate tools strategically. (MP)

Content Area: Mathematics
Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Grade Level Expectation: Eighth Grade

Concepts and skills students master:

3. Graphs, tables and equations can be used to distinguish between linear and nonlinear functions

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Define, evaluate, and compare functions. (CCSS: 8.F) i. Define a function as a rule that assigns to each input exactly one output.[5] (CCSS: 8.F.1) ii. Show that the graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (CCSS: 8.F.1) iii. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).[6] (CCSS: 8.F.2) iv. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line. (CCSS: 8.F.3) v. Give examples of functions that are not linear.[7] b. Use functions to model relationships between quantities. (CCSS: 8.F) i. Construct a function to model a linear relationship between two quantities. (CCSS: 8.F.4) ii. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. (CCSS: 8.F.4) iii. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. (CCSS: 8.F.4) iv. Describe qualitatively the functional relationship between two quantities by analyzing a graph.[8] (CCSS: 8.F.5) v. Sketch a graph that exhibits the qualitative features of a function that has been described verbally. (CCSS: 8.F.5) vi. Analyze how credit and debt impact personal financial goals (PFL)	**Inquiry Questions:** 1. How can change best be represented mathematically? 2. Why are patterns and relationships represented in multiple ways? 3. What properties of a function make it a linear function? **Relevance and Application:** 1. Recognition that non-linear situations is a clue to non-constant growth over time helps to understand such concepts as compound interest rates, population growth, appreciations, and depreciation. 2. Linear situations allow for describing and analyzing the situation mathematically such as using a line graph to represent the relationships of the circumference of circles based on diameters. **Nature of Mathematics:** 1. Mathematics involves multiple points of view. 2. Mathematicians look at mathematical ideas arithmetically, geometrically, analytically, or through a combination of these approaches. 3. Mathematicians look for and make use of structure. (MP) 4. Mathematicians look for and express regularity in repeated reasoning. (MP)

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[1] For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed. (CCSS: 8.EE.5)

[2] Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). (CCSS: 8.EE.6a)

[3] For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6. (CCSS: 8.EE.8b)

[4] For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair. (CCSS: 8.EE.8c)

[5] Function notation is not required in 8th grade. (CCSS: 8.F.1[1])

[6] For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change. (CCSS: 8.F.2)

[7] For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line. (CCSS: 8.F.3)

[8] e.g., where the function is increasing or decreasing, linear or nonlinear. (CCSS: 8.F.5)

3. Data Analysis, Statistics, and Probability

Data and probability sense provides students with tools to understand information and uncertainty. Students ask questions and gather and use data to answer them. Students use a variety of data analysis and statistics strategies to analyze, develop and evaluate inferences based on data. Probability provides the foundation for collecting, describing, and interpreting data.

Prepared Graduates

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the 3. Data Analysis, Statistics, and Probability Standard are:

- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts
- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data
- Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Content Area: Mathematics
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

Grade Level Expectation: Eighth Grade

Concepts and skills students master:

1. Visual displays and summary statistics of two-variable data condense the information in data sets into usable knowledge

Evidence Outcomes

Students can:

a. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. (CCSS: 8.SP.1)
b. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. (CCSS: 8.SP.1)
c. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.[1] (CCSS: 8.SP.2)
d. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.[2] (CCSS: 8.SP.3)
e. Explain patterns of association seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. (CCSS: 8.SP.4)
    i. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. (CCSS: 8.SP.4)
    ii. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.[3] (CCSS: 8.SP.4)

21st Century Skills and Readiness Competencies

Inquiry Questions:

1. How is it known that two variables are related to each other?
2. How is it known that an apparent trend is just a coincidence?
3. How can correct data lead to incorrect conclusions?
4. How do you know when a credible prediction can be made?

Relevance and Application:

1. The ability to analyze and interpret data helps to distinguish between false relationships such as developing superstitions from seeing two events happen in close succession versus identifying a credible correlation.
2. Data analysis provides the tools to use data to model relationships, make predictions, and determine the reasonableness and limitations of those predictions. For example, predicting whether staying up late affects grades, or the relationships between education and income, between income and energy consumption, or between the unemployment rate and GDP.

Nature of Mathematics:

1. Mathematicians discover new relationship embedded in information.
2. Mathematicians construct viable arguments and critique the reasoning of others. (MP)
3. Mathematicians model with mathematics. (MP)

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[1] Know that straight lines are widely used to model relationships between two quantitative variables. (CCSS: 8.SP.2)
[2] For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height. (CCSS: 8.SP.3)
[3] For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores? (CCSS: 8.SP.4)

4. Shape, Dimension, and Geometric Relationships

Geometric sense allows students to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. Students learn that geometry and measurement are useful in representing and solving problems in the real world as well as in mathematics.

Prepared Graduates
The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the 4. Shape, Dimension, and Geometric Relationships standard are:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Apply transformation to numbers, shapes, functional representations, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Apply transformation to numbers, shapes, functional representations, and data

Grade Level Expectation: Eighth Grade

Concepts and skills students master:

1. Transformations of objects can be used to define the concepts of congruence and similarity

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Verify experimentally the properties of rotations, reflections, and translations.[1] (CCSS: 8.G.1) b. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. (CCSS: 8.G.3) c. Demonstrate that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations. (CCSS: 8.G.2) d. Given two congruent figures, describe a sequence of transformations that exhibits the congruence between them. (CCSS: 8.G.2) e. Demonstrate that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations. (CCSS: 8.G.4) f. Given two similar two-dimensional figures, describe a sequence of transformations that exhibits the similarity between them. (CCSS: 8.G.4) g. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.[2] (CCSS: 8.G.5)	**Inquiry Questions:** 1. What advantage, if any, is there to using the Cartesian coordinate system to analyze the properties of shapes? 2. How can you physically verify that two lines are really parallel? **Relevance and Application:** 1. Dilations are used to enlarge or shrink pictures. 2. Rigid motions can be used to make new patterns for clothing or architectural design. **Nature of Mathematics:** 1. Geometry involves the investigation of invariants. Geometers examine how some things stay the same while other parts change to analyze situations and solve problems. 2. Mathematicians construct viable arguments and critique the reasoning of others. (MP) 3. Mathematicians model with mathematics. (MP)

Content Area: Mathematics
Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Grade Level Expectation: Eighth Grade

Concepts and skills students master:

2. Direct and indirect measurement can be used to describe and make comparisons

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Explain a proof of the Pythagorean Theorem and its converse. (CCSS: 8.G.6) b. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. (CCSS: 8.G.7) c. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. (CCSS: 8.G.8) d. State the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems. (CCSS: 8.G.9)	**Inquiry Questions:** 1. Why does the Pythagorean Theorem only apply to right triangles? 2. How can the Pythagorean Theorem be used for indirect measurement? 3. How are the distance formula and the Pythagorean theorem the same? Different? 4. How are the volume formulas for cones, cylinders, prisms and pyramids interrelated? 5. How is volume of an irregular figure measured? 6. How can cubic units be used to measure volume for curved surfaces? **Relevance and Application:** 1. The understanding of indirect measurement strategies allows measurement of features in the immediate environment such as playground structures, flagpoles, and buildings. 2. Knowledge of how to use right triangles and the Pythagorean Theorem enables design and construction of such structures as a properly pitched roof, handicap ramps to meet code, structurally stable bridges, and roads. 3. The ability to find volume helps to answer important questions such as how to minimize waste by redesigning packaging or maximizing volume by using a circular base. **Nature of Mathematics:** 1. Mathematicians use geometry to model the physical world. Studying properties and relationships of geometric objects provides insights in to the physical world that would otherwise be hidden. 2. Geometric objects are abstracted and simplified versions of physical objects 3. Mathematicians make sense of problems and persevere in solving them. (MP) 4. Mathematicians construct viable arguments and critique the reasoning of others. (MP)

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[1] Lines are taken to lines, and line segments to line segments of the same length. (CCSS: 8.G.1a)
Angles are taken to angles of the same measure. (CCSS: 8.G.1b)
Parallel lines are taken to parallel lines. (CCSS: 8.G.1c)

[2] For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so. (CCSS: 8.G.5)

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