Colorado
Academic Standards

Mathematics

COLORADO
Department of Education

ALL STUDENTS • ALL STANDARDS

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Purpose of Mathematics

"Pure mathematics is, in its way, the poetry of logical ideas."

~Albert Einstein, *Obituary for Emmy Noether* (1935)

"Systematization is a great virtue of mathematics, and if possible, the student has to learn this virtue, too. But then I mean the activity of systematizing, not its result. Its result is a system, a beautiful closed system, closed with no entrance and no exit. In its highest perfection it can even be handled by a machine. But for what can be performed by machines, we need no humans. What humans have to learn is not mathematics as a closed system, but rather as an activity, the process of mathematizing reality and if possible even that of mathematizing mathematics."

~Hans Freudenthal, *Why to Teach Mathematics So as to Be Useful* (1968)

Mathematics is the human activity of reasoning with number and shape, in concert with the logical and symbolic artifacts that people develop and apply in their mathematical activity. The National Council of Teachers of Mathematics (2018) outlines three primary purposes for learning mathematics:

1. To Expand Professional Opportunity. Just as the ability to read and write was critical for workers when the early 20th century economy shifted from agriculture to manufacturing, the ability to do mathematics is critical for workers in the 21st-century as the economy has shifted from manufacturing to information technology. Workers with a robust understanding of mathematics are in demand by employers, and job growth in STEM (science, technology, engineering, and mathematics) fields is forecast to accelerate over the next decade.

2. Understand and Critique the World. A consequence of living in a technological society is the need to interpret and understand the mathematics behind our social, scientific, commercial, and political systems. Much of this mathematics appears in the way of statistics, tables, and graphs, but this need to understand and critique the world extends to the application of mathematical models, attention given to precision, bias in data collection, and the soundness of mathematical claims and arguments. Learners of mathematics should feel empowered to make sense of the world around them and to better participate as an informed member of a democratic society.

3. Experience Wonder, Joy, and Beauty. Just as human forms and movement can be beautiful in dance, or sounds can make beautiful music, the patterns, shapes, and reasoning of mathematics can also be beautiful. On a personal level, mathematical problem solving can be an authentic act of individual creativity, while on a societal level, mathematics both informs and is informed by the culture of those who use and develop it, just as art or language is used and developed.

References

National Council of Teachers of Mathematics (2018). *Catalyzing change in high school mathematics: Initiating critical conversations*. Reston, VA: National Council of Teachers of Mathematics.

Prepared Graduates in Mathematics

Prepared graduates in mathematics are described by the eight *Standards for Mathematical Practice* described in the Common Core State Standards:

MP1. Make sense of problems and persevere in solving them.

MP2. Reason abstractly and quantitatively.

MP3. Construct viable arguments and critique the reasoning of others.

MP4. Model with mathematics.

MP5. Use appropriate tools strategically.

MP6. Attend to precision.

MP7. Look for and make use of structure.

MP8. Look for and express regularity in repeated reasoning.

Standards in Mathematics

The Colorado Academic Standards in mathematics are the topical organization of the concepts and skills every Colorado student should know and be able to do throughout their preschool through twelfth grade experience. The standards of mathematics are:

1. Number and Quantity
From preschool through high school, students are continually extending their concept of numbers as they build an understanding of whole numbers, rational numbers, real numbers, and complex numbers. As they engage in real-world mathematical problems, they conceive of quantities, numbers with associated units. Students learn that numbers are governed by properties and understand these properties lead to fluency with operations.

2. Algebra and Functions
Algebraic thinking is about understanding and using numbers, and students' work in this area helps them extend the arithmetic of early grades to expressions, equations, and functions in later grades. This mathematics is applied to real-world problems as students use numbers, expressions, and equations to model the world. The mathematics of this standard is closely related to that of Number and Quantity.

3. Data Analysis, Statistics, and Probability
From the early grades, students gather, display, summarize, examine, and interpret data to discover patterns and deviations from patterns. Measurement is used to generate, represent and analyze data. Working with data and an understanding of the principles of probability lead to a formal study of statistics in middle in high school. Statistics provides tools for describing variability in data and for making informed decisions that take variability into account.

4. Geometry
Students' study of geometry allows them to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, and engage in logical reasoning. Students learn that geometry is useful in representing, modeling, and solving problems in the real world as well as in mathematics.

Modeling Across the High School Standards

A star symbol (★) in the high school standards represents grade level expectations and evidence outcomes that make up a mathematical modeling standards category.

Modeling links classroom mathematics and statistics to everyday life, work, and decision making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards.

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Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP3. Construct viable arguments and critique the reasoning of others.
MP7. Look for and make use of structure.

Grade Level Expectation:

6.RP.A. Ratios & Proportional Relationships: Understand ratio concepts and use ratio reasoning to solve problems.

Evidence Outcomes

Students Can:

1. Apply the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was* $2:1$*, because for every* 2 *wings there was* 1 *beak." "For every vote Candidate* A *received, Candidate* C *received nearly three votes."* (CCSS: 6.RP.A.1)
2. Apply the concept of a unit rate $\frac{a}{b}$ associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of* 3 *cups of flour to* 4 *cups of sugar, so there is* $\frac{3}{4}$ *cup of flour for each cup of sugar." "We paid $75 for* 15 *hamburgers, which is a rate of $5 per hamburger." (Expectations for unit rates in this grade are limited to non-complex fractions.)* (CCSS: 6.RP.A.2)
3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. (CCSS: 6.RP.A.3)
   a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. (CCSS: 6.RP.A.3.a)
   b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took* 7 *hours to mow* 4 *lawns, then at that rate, how many lawns could be mowed in* 35 *hours? At what rate were lawns being mowed?* (CCSS: 6.RP.A.3.b)
   c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means $\frac{30}{100}$ times the quantity); solve problems involving finding the whole, given a part and the percent. (CCSS: 6.RP.A.3.c)
   d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. (CCSS: 6.RP.A.3.d)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Use ratio tables to test solutions and determine equivalent ratios. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Analyze and use appropriate quantities and pay attention to units in problems that require reasoning with ratios. (MP2)
3. Construct arguments that compare quantities using ratios or rates. (MP3)
4. Use tables, tape diagrams, and double number line diagrams to provide a structure for seeing equivalency between ratios. (MP7)

Inquiry Questions:

1. How are ratios different from fractions?
2. What is the difference between a quantity and a number?
3. How is a percent also a ratio?
4. How is a rate similar to and also different from a unit rate?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In prior grades, students work with multiplication, division, and measurement. Prior knowledge with the structure of the multiplication table is an important connection for students in creating and verifying equivalent ratios written in symbolic form or in ratio tables (multiplicative comparison vs. additive comparison).
3. In Grade 6, this expectation connects with one-variable equations, inequalities, and representing and analyzing quantitative relationships between dependent and independent variables.
4. In Grade 7, students analyze proportional relationships and use them to solve real-world and mathematical problems. In high school, students generalize rates of change to linear and nonlinear functions and use them to describe real-world scenarios.

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Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.

Grade Level Expectation:

6.NS.A. The Number System: Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

Evidence Outcomes

Students Can:

1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for* $\frac{2}{3} \div \frac{3}{4}$ *and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that* $\frac{2}{3} \div \frac{3}{4} = \frac{8}{9}$ *because* $\frac{3}{4}$ *of* $\frac{8}{9}$ *is* $\frac{2}{3}$*. (In general,* $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$*.) How much chocolate will each person get if 3 people share* $\frac{1}{2}$ *lb of chocolate equally? How many* $\frac{3}{4}$*-cup servings are in* $\frac{2}{3}$ *of a cup of yogurt? How wide is a rectangular strip of land with length* $\frac{3}{4}$ *mi and area* $\frac{1}{2}$ *square mi?* (CCSS: 6.NS.A.1)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Create and solve word problems using division of fractions, understanding the relationship of the arithmetic to the problem being solved. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Reason about the contextualized meaning of numbers in word problems involving division of fractions, and decontextualize those numbers to perform efficient calculations. (MP2)
3. Model real-world situations involving scaling by non-whole numbers using multiplication and division by fractions. (MP4)

Inquiry Questions:

1. When dividing, is the quotient always going to be a smaller number than the dividend? Why or why not?
2. What kinds of real-world situations require the division of fractions?
3. How can the division of fractions be modeled visually?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 5, students apply and extend previous understandings of multiplication and division to divide whole numbers by unit fractions and unit fractions by whole numbers.
3. In Grade 6, this expectation connects with solving one-step, one-variable equations and inequalities.
4. In Grade 7, students apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

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Prepared Graduates:

MP6. Attend to precision.
MP7. Look for and make use of structure.

Grade Level Expectation:

6.NS.B. The Number System: Compute fluently with multi-digit numbers and find common factors and multiples.

Evidence Outcomes

Students Can:

2. Fluently divide multi-digit numbers using the standard algorithm. (CCSS: 6.NS.B.2)
3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. (CCSS: 6.NS.B.3)
4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express* $36 + 8$ *as* $4(9 + 2)$. (CCSS: 6.NS.B.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Accurately add, subtract, multiply, and divide with decimals. (MP6)
2. Recognize the structures of factors and multiples when identifying the greatest common factor and least common multiple of two whole numbers. Use the greatest common factor to rewrite an expression using the distributive property. (MP7)

Inquiry Questions:

1. How do operations with decimals compare and contrast to operations with whole numbers?
2. How does rewriting the sum of two whole numbers using the distributive property yield new understanding and insights on the sum?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In Grade 5, students divide whole numbers with two-digit divisors and perform operations with decimals.
3. In Grade 6, this expectation connects with applying and extending previous understandings of arithmetic to algebraic expressions.
4. In Grade 7, students apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. Students apply the concept of greatest common factor to factor linear expressions, and extending properties of whole numbers to variable expressions.

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Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP3. Construct viable arguments and critique the reasoning of others.
MP5. Use appropriate tools strategically.

Grade Level Expectation:

6.NS.C. The Number System: Apply and extend previous understandings of numbers to the system of rational numbers.

Evidence Outcomes

Students Can:

5. Explain why positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. (CCSS: 6.NS.C.5)
6. Describe a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. (CCSS: 6.NS.C.6)
   a. Use opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; identify that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite. (CCSS: 6.NS.C.6.a)
   b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; explain that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. (CCSS: 6.NS.C.6.b)
   c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. (CCSS: 6.NS.C.6.c)
7. Order and find absolute value of rational numbers. (CCSS: 6.NS.C.7)
   a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret* $-3 > -7$ *as a statement that* -3 *is located to the right of* -7 *on a number line oriented from left to right.* (CCSS: 6.NS.C.7.a)
   b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write* $-3°C > -7°C$ *to express the fact that* $-3°C$ *is warmer than* $-7°C$. (CCSS: 6.NS.C.7.b)
   c. Define the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of* -30 *dollars, write* $|-30| = 30$ *to describe the size of the debt in dollars.* (CCSS: 6.NS.C.7.c)
   d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than* -30 *dollars represents a debt greater than* 30 *dollars.* (CCSS: 6.NS.C.7.d)
8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. (CCSS: 6.NS.C.8)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Investigate integers to form hypotheses, make observations and draw conclusions. (Entrepreneurial Skills: Inquiry/Analysis)
2. Understand the relationship among negative numbers, positive numbers, and absolute value. (MP2)
3. Explain the order of rational numbers using their location on the number line. (MP3)
4. Demonstrate how to plot points on a number line and plot ordered pairs on a coordinate plane. (MP5)

Inquiry Questions:

1. Why do we have negative numbers?
2. What relationships exist among positive and negative numbers on the number line?
3. How does the opposite of a number differ from the absolute value of that same number?
4. How does an ordered pair correspond to its given point on a coordinate plane?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In previous grades, students develop understanding of fractions as numbers and graph points on the coordinate plane (limited to the first quadrant) to solve real-world and mathematical problems.
3. In Grade 6, this expectation connects with reasoning about and solving one-step, one-variable equations and inequalities.
4. In Grade 7, students apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. In Grade 8, students investigate patterns of association in bivariate data.

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Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP7. Look for and make use of structure.
MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

6.EE.A. Expressions & Equations: Apply and extend previous understandings of arithmetic to algebraic expressions.

Evidence Outcomes

Students Can:

1. Write and evaluate numerical expressions involving whole-number exponents. (CCSS: 6.EE.A.1)
2. Write, read, and evaluate expressions in which letters stand for numbers. (CCSS: 6.EE.A.2)
   a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as $5 - y$.* (CCSS: 6.EE.A.2.a)
   b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression $2(8 + 7)$ as a product of two factors; view $(8 + 7)$ as both a single entity and a sum of two terms.* (CCSS: 6.EE.A.2.b)
   c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = \frac{1}{2}$.* (CCSS: 6.EE.A.2.c)
3. Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6(4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.* (CCSS: 6.EE.A.3)
4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.* (CCSS: 6.EE.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Recognize that expressions can be written in multiple forms and describe cause-and-effect relationships and patterns. (Entrepreneurial Skills: Critical Thinking/Problem Solving and Inquiry/Analysis)
2. Communicate a justification of why expressions are equivalent using arguments about properties of operations and whole numbers. (MP3)
3. See the structure of an expression like $x + 2$ as a sum but also as a single factor in the product $3(x + 2)$. (MP7)
4. Recognize equivalence in variable expressions with repeated addition (such as $y + y + y = 3y$) and repeated multiplication (such as $y \times y \times y = y^3$) and use arithmetic operations to justify the equivalence. (MP8)

Inquiry Questions:

1. How are algebraic expressions similar to and different from numerical expressions?
2. What does it mean for two variable expressions to be equivalent?
3. How might the application of the order of operations differ when using grouping symbols, such as parentheses, for numerical expressions as compared to algebraic expressions?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In previous grades, students understand and apply properties of operations, relationships between inverse arithmetic operations, and write and interpret numerical expressions.
3. In Grade 6, this expectation connects to fluency with multi-digit numbers, finding common factors and multiples, and one-variable equations and inequalities.
4. In future grades, students work with radicals and integer exponents and interpret the structure of more complex algebraic expressions.

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Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP6. Attend to precision.

Grade Level Expectation:

6.EE.B. Expressions & Equations: Reason about and solve one-variable equations and inequalities.

Evidence Outcomes

Students Can:

5. Describe solving an equation or inequality as a process of answering a question: Which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. (CCSS: 6.EE.B.5)
6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; recognize that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. (CCSS: 6.EE.B.6)
7. Solve real-world and mathematical problems by writing and solving equations of the form $x \pm p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. (CCSS: 6.EE.B.7)
8. Write an inequality of the form $x > c$, $x \geq c$, $x < c$, or $x \leq c$ to represent a constraint or condition in a real-world or mathematical problem. Show that inequalities of the form $x > c$, $x \geq c$, $x < c$, or $x \leq c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams. (CCSS: 6.EE.B.8)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Investigate unknown values to form hypotheses, make observations, and draw conclusions. (Entrepreneurial Skills: Inquiry/Analysis)
2. Reason about the values and operations of an equation both within a real-world context and abstracted from it. (MP2)
3. State precisely the meaning of variables used when setting up equations. (MP6)

Inquiry Questions:

1. What are the different ways a variable can be used in an algebraic equation or inequality? For example, how are these uses of the variable x different from each other? (a) $x + 5 = 8$; (b) $x = \frac{1}{2}$; (c) $x > 5$.
2. How is the solution to an inequality different than a solution to an equation?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In previous grades, students write simple expressions that record calculations with numbers, interpret numerical expressions without evaluating them, and generate ordered pairs from two numerical rules.
3. This expectation connects to several others in Grade 6: (a) understanding ratio concepts and use ratio reasoning to solve problems, (b) applying and extending previous understandings of multiplication and division to divide fractions by fractions, (c) applying and extending previous understandings of numbers to the system of rational numbers, (d) applying and extending previous understandings of arithmetic to algebraic expressions, and (e) representing and analyzing quantitative relationships between dependent and independent variables.
4. In Grade 7, students solve real-life and mathematical problems involving two-step equations and inequalities. In Grade 8, students work with radicals and integer exponents and solve linear equations and pairs of simultaneous linear equations.

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Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.

Grade Level Expectation:

6.EE.C. Expressions & Equations: Represent and analyze quantitative relationships between dependent and independent variables.

Evidence Outcomes

Students Can:

9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation* $d = 65t$ *to represent the relationship between distance and time.* (CCSS: 6.EE.C.9)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Analyze relationships between dependent and independent variables.
2. Reason about the operations that relate constant and variable quantities in equations with dependent and independent variables. (MP2)
3. Model with mathematics by describing real-world situations with equations and inequalities. (MP4)

Inquiry Questions:

1. How can you determine if a variable is the independent variable or the dependent variable?
2. What are the advantages of showing the relationship between an independent and dependent variable in multiple representations (table, graph, equation)?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 5, students analyze numerical patterns and relationships, including generating and graphing ordered pairs in the first quadrant.
3. In Grade 6, this expectation connects with understanding ratio concepts and using ratio reasoning to solve problems.
4. In Grade 7, students decide if two quantities are in a proportional relationship and identify the unit rate in tables, graphs, equations, diagrams, and verbal descriptions.

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Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.

Grade Level Expectation:

6.SP.A. Statistics & Probability: Develop understanding of statistical variability.

Evidence Outcomes

Students Can:

1. Identify a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.* (CCSS: 6.SP.A.1)
2. Demonstrate that a set of data collected to answer a statistical question has a distribution that can be described by its center, spread, and overall shape. (CCSS: 6.SP.A.2)
3. Explain that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. (CCSS: 6.SP.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Identify statistical questions that require the collection of data representing multiple perspectives. (Civic/Interpersonal Skills: Global/Cultural Awareness)
2. Make sense of practical problems by turning them into statistical investigations. (MP1)
3. Reason abstractly and quantitatively with data collected from statistical investigations by describing the data's center, spread, and shape.
4. Model variability in data collected to answer statistical questions and draw conclusions based on center, spread, and shape. (MP4)

Inquiry Questions:

1. What distinguishes a statistical question from a question that is not a statistical question?
2. Why do we have numerical measures for both the center of a data set and the variability of a data set?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In previous grades, students represent and interpret data in dot plots/line plots, and use arithmetic to answer questions about the plots.
3. In Grade 6, students summarize and describe data distributions using numerical measures of center and spread, and terms such as cluster, peak, gap, symmetry, skew, and outlier.
4. In Grade 7, students use random sampling to draw inferences about a population and draw informal comparative inferences about two populations. In high school, students summarize, represent, and interpret data on a single count or measurement variable.

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Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.
MP7. Look for and make use of structure.

Grade Level Expectation:

6.SP.B. Statistics & Probability: Summarize and describe distributions.

Evidence Outcomes

Students Can:

4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots. (CCSS: 6.SP.B.4)
5. Summarize numerical data sets in relation to their context, such as by: (CCSS: 6.SP.B.5)
   a. Reporting the number of observations. (CCSS: 6.SP.B.5.a)
   b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. (CCSS: 6.SP.B.5.b)
   c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. (CCSS: 6.SP.B.5.c)
   d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered. (CCSS: 6.SP.B.5.d)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Write informative texts describing statistical distributions, their measures, and how they relate to the context in which the data were gathered. (Entrepreneurial Skills: Literacy/Writing)
2. Move from context to abstraction and back to context while finding and using measures of center and variability and describing what they mean in the context of the data. (MP2)
3. Analyze data sets and draw conclusions based on the data display and measures of center and/or variability. (MP4)
4. Identify clusters, peaks, gaps, and symmetry, and describe the meaning of these and other patterns in data distributions. (MP7)

Inquiry Questions:

1. How can different data displays communicate different meanings?
2. When is it better to use the mean as a measure of center? Why?
3. When is it better to use the median as a measure of center? Why?
4. How many values of a data set do you use to calculate the range? Interquartile range? Mean absolute deviation? How does this help to compare what these measures represent?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In previous grades, students represent and interpret data in dot plots and line plots.
3. In Grade 6, this expectation connects with developing understanding of statistical variability.
4. In high school, students summarize, represent, and interpret data on a single count or measurement variable (including standard deviation), and make inferences and justify conclusions from sample surveys, experiments, and observational studies.

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Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP4. Model with mathematics.
MP5. Use appropriate tools strategically.

Grade Level Expectation:

6.G.A. Geometry: Solve real-world and mathematical problems involving area, surface area, and volume.

Evidence Outcomes

Students Can:

1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. (CCSS: 6.G.A.1)
2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. (CCSS: 6.G.A.2)
3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. (CCSS: 6.G.A.3)
4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems. (CCSS: 6.G.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Recognize that problems can be identified and possible solutions can be created with respect to using area, surface area, and volume. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Make sense of a problem by understanding the context of the problem before applying a formula. (MP1)
3. Model real-world problems involving shape and space. (MP4)
4. Strategically use coordinate planes, nets of three-dimensional figures, and area and volume formulas as tools to solve real-world problems. (MP5)

Inquiry Questions:

1. What is the difference between what area measures and what volume measures?
2. How does using decomposition aid in finding the area of composite figures?
3. How are nets of three-dimensional figures used to find surface area?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In Grade 4, students solve problems involving measurement and converting from a larger unit to a smaller unit. In Grade 5, students understand concepts of volume, relate volume to multiplication and to addition, and graph points on the coordinate plane to solve real-world and mathematical problems.
3. In Grade 6, this expectation connects with graphing points in all four quadrants of the coordinate plane and finding distances between points with the same first coordinate or the same second coordinate.
4. In Grade 7, students draw, construct, and describe geometrical figures and describe the relationships between them, and solve real-world and mathematical problems involving angle measure, area, surface area, and volume. In Grade 8, students understand congruence and similarity and understand and apply the Pythagorean Theorem.