Colorado
Academic Standards

Mathematics

COLORADO
Department of Education

ALL STUDENTS • ALL STANDARDS

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Purpose of Mathematics

"Pure mathematics is, in its way, the poetry of logical ideas."

~Albert Einstein, *Obituary for Emmy Noether* (1935)

"Systematization is a great virtue of mathematics, and if possible, the student has to learn this virtue, too. But then I mean the activity of systematizing, not its result. Its result is a system, a beautiful closed system, closed with no entrance and no exit. In its highest perfection it can even be handled by a machine. But for what can be performed by machines, we need no humans. What humans have to learn is not mathematics as a closed system, but rather as an activity, the process of mathematizing reality and if possible even that of mathematizing mathematics."

~Hans Freudenthal, *Why to Teach Mathematics So as to Be Useful* (1968)

Mathematics is the human activity of reasoning with number and shape, in concert with the logical and symbolic artifacts that people develop and apply in their mathematical activity. The National Council of Teachers of Mathematics (2018) outlines three primary purposes for learning mathematics:

1. To Expand Professional Opportunity. Just as the ability to read and write was critical for workers when the early 20th century economy shifted from agriculture to manufacturing, the ability to do mathematics is critical for workers in the 21st-century as the economy has shifted from manufacturing to information technology. Workers with a robust understanding of mathematics are in demand by employers, and job growth in STEM (science, technology, engineering, and mathematics) fields is forecast to accelerate over the next decade.

2. Understand and Critique the World. A consequence of living in a technological society is the need to interpret and understand the mathematics behind our social, scientific, commercial, and political systems. Much of this mathematics appears in the way of statistics, tables, and graphs, but this need to understand and critique the world extends to the application of mathematical models, attention given to precision, bias in data collection, and the soundness of mathematical claims and arguments. Learners of mathematics should feel empowered to make sense of the world around them and to better participate as an informed member of a democratic society.

3. Experience Wonder, Joy, and Beauty. Just as human forms and movement can be beautiful in dance, or sounds can make beautiful music, the patterns, shapes, and reasoning of mathematics can also be beautiful. On a personal level, mathematical problem solving can be an authentic act of individual creativity, while on a societal level, mathematics both informs and is informed by the culture of those who use and develop it, just as art or language is used and developed.

References

National Council of Teachers of Mathematics (2018). *Catalyzing change in high school mathematics: Initiating critical conversations*. Reston, VA: National Council of Teachers of Mathematics.

Prepared Graduates in Mathematics

Prepared graduates in mathematics are described by the eight *Standards for Mathematical Practice* described in the Common Core State Standards:

MP1. Make sense of problems and persevere in solving them.

MP2. Reason abstractly and quantitatively.

MP3. Construct viable arguments and critique the reasoning of others.

MP4. Model with mathematics.

MP5. Use appropriate tools strategically.

MP6. Attend to precision.

MP7. Look for and make use of structure.

MP8. Look for and express regularity in repeated reasoning.

Standards in Mathematics

The Colorado Academic Standards in mathematics are the topical organization of the concepts and skills every Colorado student should know and be able to do throughout their preschool through twelfth grade experience. The standards of mathematics are:

1. Number and Quantity
From preschool through high school, students are continually extending their concept of numbers as they build an understanding of whole numbers, rational numbers, real numbers, and complex numbers. As they engage in real-world mathematical problems, they conceive of quantities, numbers with associated units. Students learn that numbers are governed by properties and understand these properties lead to fluency with operations.

2. Algebra and Functions
Algebraic thinking is about understanding and using numbers, and students' work in this area helps them extend the arithmetic of early grades to expressions, equations, and functions in later grades. This mathematics is applied to real-world problems as students use numbers, expressions, and equations to model the world. The mathematics of this standard is closely related to that of Number and Quantity.

3. Data Analysis, Statistics, and Probability
From the early grades, students gather, display, summarize, examine, and interpret data to discover patterns and deviations from patterns. Measurement is used to generate, represent and analyze data. Working with data and an understanding of the principles of probability lead to a formal study of statistics in middle in high school. Statistics provides tools for describing variability in data and for making informed decisions that take variability into account.

4. Geometry
Students' study of geometry allows them to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, and engage in logical reasoning. Students learn that geometry is useful in representing, modeling, and solving problems in the real world as well as in mathematics.

Modeling Across the High School Standards

A star symbol (★) in the high school standards represents grade level expectations and evidence outcomes that make up a mathematical modeling standards category.

Modeling links classroom mathematics and statistics to everyday life, work, and decision making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards.

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Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

7.RP.A. Ratios & Proportional Relationships: Analyze proportional relationships and use them to solve real-world and mathematical problems.

Evidence Outcomes

Students Can:

1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. *For example, if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\frac{\frac{1}{2}}{\frac{1}{4}}$ miles per hour, equivalently 2 miles per hour.* (CCSS: 7.RP.A.1)
2. Identify and represent proportional relationships between quantities. (CCSS: 7.RP.A.2)
   a. Determine whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. (CCSS: 7.RP.A.2.a)
   b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. (CCSS: 7.RP.A.2.b)
   c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.* (CCSS: 7.RP.A.2.c)
   d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0,0)$ and $(1, r)$ where r is the unit rate. (CCSS: 7.RP.A.2.d)
3. Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.* (CCSS: 7.RP.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Recognize when proportional relationships occur and apply these relationships to personal experiences. (Entrepreneurial Skills: Inquiry/Analysis)
2. Recognize, identify, and solve problems that involve proportional relationships to make predictions and describe associations among variables. (MP1)
3. Reason quantitatively with rates and their units in proportional relationships. (MP2)
4. Use repeated reasoning to test for equivalent ratios, such as reasoning that walking $\frac{1}{2}$ mile in $\frac{1}{4}$ hour is equivalent to walking 1 mile in $\frac{1}{2}$ hour and equivalent to walking 2 miles in 1 hour, the unit rate. (MP8)

Inquiry Questions:

1. How are proportional relationships related to unit rates?
2. How can proportional relationships be expressed using tables, equations, and graphs?
3. What are properties of all proportional relationships when graphed on the coordinate plane?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 6, students understand ratio concepts and use ratio reasoning to solve problems.
3. This expectation connects with several others in Grade 7: (a) solving real-life and mathematical problems using numerical and algebraic expressions and equations, (b) investigating chance processes and developing, using, and evaluating probability models, and (c) drawing, constructing, and describing geometrical figures and describing the relationships between them.
4. In Grade 8, students (a) understand the connections between proportional relationships, lines, and linear equations, (b) define, evaluate, and compare functions, and (c) use functions to model relationships between quantities. In high school, students use proportional relationships to define trigonometric ratios, solve problems involving right triangles, and find arc lengths and areas of sectors of circles.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP3. Construct viable arguments and critique the reasoning of others.
MP7. Look for and make use of structure.

Grade Level Expectation:

7.NS.A. The Number System: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Evidence Outcomes

Students Can:

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. (CCSS: 7.NS.A.1)
   a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.* (CCSS: 7.NS.A.1.a)
   b. Demonstrate $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. (CCSS: 7.NS.A.1.b)
   c. Demonstrate subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. (CCSS: 7.NS.A.1.c)
   d. Apply properties of operations as strategies to add and subtract rational numbers. (CCSS: 7.NS.A.1.d)
2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. (CCSS: 7.NS.A.2)
   a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. (CCSS: 7.NS.A.2.a)
   b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-\left(\frac{p}{q}\right) = \frac{-p}{q} = \frac{p}{-q}$. Interpret quotients of rational numbers by describing real-world contexts. (CCSS: 7.NS.A.2.b)
   c. Apply properties of operations as strategies to multiply and divide rational numbers. (CCSS: 7.NS.A.2.c)
   d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. (CCSS: 7.NS.A.2.d)
3. Solve real-world and mathematical problems involving the four operations with rational numbers. *(Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)* (CCSS: 7.NS.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Solve problems with rational numbers using all four operations. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Compute with rational numbers abstractly and interpret quantities in context. (MP2)
3. Justify understanding and computational accuracy of operations with rational numbers. (MP3)
4. Use additive inverses, absolute value, the distributive property, and properties of operations to reason with and operate on rational numbers. (MP7)

Inquiry Questions:

1. How do operations with integers compare to and contrast with operations with whole numbers?
2. How can operations with negative integers be modeled visually?
3. How can it be determined if the decimal form of a rational number terminates or repeats?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In previous grades, students use the four operations with whole numbers and fractions to solve problems.
3. In Grade 7, this expectation connects with solving real-life and mathematical problems using numerical and algebraic expressions and equations. This expectation begins the formal study of rational numbers (a number expressible in the form $\frac{a}{b}$ or $-\frac{a}{b}$ for some fraction $\frac{a}{b}$; the rational numbers include the integers) as extended from their study of fractions, which in these standards always refers to non-negative numbers.
4. In Grade 8, students extend their study of the real number system to include irrational numbers, radical expressions, and integer exponents. In high school, students work with rational exponents and complex numbers.

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Prepared Graduates:

MP7. Look for and make use of structure.

Grade Level Expectation:

7.EE.A. Expressions & Equations: Use properties of operations to generate equivalent expressions.

Evidence Outcomes

Students Can:

1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. (CCSS: 7.EE.A.1)
2. Demonstrate that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example,* $a + 0.05a = 1.05a$ *means that "increase by* 5%*" is the same as "multiply by* 1.05*."* (CCSS: 7.EE.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Recognize that the structures of equivalent algebraic expressions provide different ways of seeing the same problem. (MP7)

Inquiry Questions:

1. How is it determined that two algebraic expressions are equivalent?
2. What is the value of having an algebraic expression in equivalent forms?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 6, students apply and extend previous understandings of arithmetic to algebraic expressions.
3. In Grade 8, students use equivalent expressions to analyze and solve linear equations and pairs of simultaneous linear equations. In high school, students use equivalent expressions within various families of functions to reveal key features of graphs and how those features are related to contextual situations.

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Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP5. Use appropriate tools strategically.
MP6. Attend to precision.

Grade Level Expectation:

7.EE.B. Expressions & Equations: Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Evidence Outcomes

Students Can:

3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making* $\$25$ *an hour gets a* 10% *raise, she will make an additional* $\frac{1}{10}$ *of her salary an hour, or* $\$2.50$*, for a new salary of* $\$27.50$*. If you want to place a towel bar* $9\frac{3}{4}$ *inches long in the center of a door that is* $27\frac{1}{2}$ *inches wide, you will need to place the bar about* 9 *inches from each edge; this estimate can be used as a check on the exact computation.* (CCSS: 7.EE.B.3)
4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. (CCSS: 7.EE.B.4)
   a. Solve word problems leading to equations of the form $px \pm q = r$ and $p(x \pm q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is* 54 *cm. Its length is* 6 *cm. What is its width?* (CCSS: 7.EE.B.4.a)
   b. Solve word problems leading to inequalities of the form $px \pm q > r$, $px \pm q \geq r$, $px \pm q < r$, or $px \pm q \leq r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid* $\$50$ *per week plus* $\$3$ *per sale. This week you want your pay to be at least* $\$100$*. Write an inequality for the number of sales you need to make and describe the solutions.* (CCSS: 7.EE.B.4.b)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Adapt to different forms of equations and inequalities and reach solutions that make sense in context. (Personal Skills: Adaptability/Flexibility)
2. Use mental computation and estimation to check the reasonableness of their solutions. Make connections between the sequence of operations used in an algebraic approach and an arithmetic approach, understanding how simply reasoning about the numbers connects to writing and solving a corresponding algebraic equation. (MP1)
3. Represent a situation symbolically and solve, attending to the meaning of quantities and variables. (MP2)
4. Select an appropriate solution approach (calculator, mental math, drawing a diagram, etc.) based on the specific values and/or desired result of a problem. (MP5)
5. Use estimation, mental calculations, and understanding of real-world contexts to assess the reasonableness of answers to real-life and mathematical problems.

Inquiry Questions:

1. Do the properties of operations apply to variables the same way they do to numbers? Why?
2. Why are there different ways to solve equations?
3. In what scenarios might estimation be better than an exact answer?
4. How can the reasonableness of a solution be determined?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 6, students reason about and solve one-step, one-variable equations and inequalities.
3. In Grade 7, this expectation connects with analyzing proportional relationships, using them to solve real-world and mathematical problems, and applying and extending previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
4. In Grade 8, students work with radicals and integer exponents, analyze and solve linear equations and pairs of simultaneous linear equations, and describe functional relationships.

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Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP4. Model with mathematics.

Grade Level Expectation:

7.SP.A. Statistics & Probability: Use random sampling to draw inferences about a population.

Evidence Outcomes

Students Can:

1. Understand that statistics can be used to gain information about a population by examining a sample of the population; explain that generalizations about a population from a sample are valid only if the sample is representative of that population. Explain that random sampling tends to produce representative samples and support valid inferences. (CCSS: 7.SP.A.1)
2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.* (CCSS: 7.SP.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Infer about a population using a random sample. (Entrepreneurial Skills: Inquiry/Analysis)
2. Make conjectures about population parameters and support arguments with sample data. (MP3)
3. Use multiple samples to informally model the variability of sample statistics like the mean. (MP4)

Inquiry Questions:

1. Why would a researcher use sampling for a study or survey?
2. Why does random sampling give more trustworthy results than nonrandom sampling in a study or survey? How might methods for obtaining a sample for a study or survey affect the results of the survey?
3. How can a winner be concluded in an election, from a sample, before counting all the ballots?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In Grade 6, students develop understanding of statistical variability.
3. In Grade 7, this expectation connects with drawing informal comparative inferences about two populations, investigating chance processes, and with developing, using, and evaluating probability models.
4. In high school, students understand and evaluate random processes underlying statistical experiments and also make inferences and justify conclusions from sample surveys, experiments, and observational studies.

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Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP4. Model with mathematics.

Grade Level Expectation:

7.SP.B. Statistics & Probability: Draw informal comparative inferences about two populations.

Evidence Outcomes

Students Can:

3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is* 10 *cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.* (CCSS: 7.SP.B.3)
4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.* (CCSS: 7.SP.B.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Interpret variability in statistical distributions and draw conclusions about the distance between their centers using units of mean absolute deviation. (Entrepreneurial Skills: Inquiry/Analysis)
2. Base arguments about the difference between two distributions on the relative variability of the distributions, not just the difference between the two distribution means. (MP3)
3. Model real-world populations with statistical distributions and compare the distributions using measures of center and variability. (MP4)

Inquiry Questions:

1. How do measures of center (such as mean) and variability (such as mean absolute deviation) work together to describe comparisons of data?
2. How can we use measures of center and variability to compare two data sets? Why is it not wise to compare two data sets using only measures of center?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In Grade 6, students study measures of center and variability to describe, compare, and contrast data sets.
3. In Grade 7, this expectation connects with using random sampling to draw inferences about a population.
4. In high school, students summarize, represent, and interpret data on a single count or measurement variable and also make inferences and justify conclusions from sample surveys, experiments, and observational studies.

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Prepared Graduates:

MP4. Model with mathematics.
MP5. Use appropriate tools strategically.

Grade Level Expectation:

7.SP.C. Statistics & Probability: Investigate chance processes and develop, use, and evaluate probability models.

Evidence Outcomes

Students Can:

5. Explain that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. (CCSS: 7.SP.C.5)
6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube* 600 *times, predict that a* 3 *or* 6 *would be rolled roughly* 200 *times, but probably not exactly* 200 *times.* (CCSS: 7.SP.C.6)
7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. (CCSS: 7.SP.C.7)
   a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* (CCSS: 7.SP.C.7.a)
   b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?* (CCSS: 7.SP.C.7.b)
8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. (CCSS: 7.SP.C.8)
   a. Explain that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. (CCSS: 7.SP.C.8.a)
   b. Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. (CCSS: 7.SP.C.8.b)
   c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If* 40% *of donors have type A blood, what is the probability that it will take at least* 4 *donors to find one with type A blood?* (CCSS: 7.SP.C.8.c)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Be innovative when designing simulations to generate frequencies of compound events by using random digits, dice, coins, or other chance objects to represent the probabilities of real-world events. (Entrepreneurial Skills: Creativity/Innovation)
2. Use probability models and simulations to predict outcomes of real-world chance events both theoretically and experimentally. (MP4)
3. Use technology, manipulatives, and simulations to determine probabilities and understand chance events. (MP5)

Inquiry Questions:

1. Since the probability of getting heads on the toss of a fair coin is $\frac{1}{2}$, does that mean for every one hundred tosses of a coin exactly fifty of them will be heads? Why or why not?
2. What might a discrepancy in the predicted outcome and the actual outcome of a chance event tell us?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In prior grades, students study rational numbers and operations with rational numbers.
3. In Grade 7, probability concepts support the major work of understanding rational numbers. This expectation connects with analyzing proportional relationships, using them to solve real-world and mathematical problems, and using random sampling to draw inferences about a population.
4. In high school, students understand and evaluate random processes underlying statistical experiments, understand independence and conditional probability and use them to interpret data, and use the rules of probability to compute probabilities of compound events in a uniform probability model.

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Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP3. Construct viable arguments and critique the reasoning of others.
MP5. Use appropriate tools strategically.

Grade Level Expectation:

7.G.A. Geometry: Draw, construct, and describe geometrical figures and describe the relationships between them.

Evidence Outcomes

Students Can:

1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. (CCSS: 7.G.A.1)
2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. (CCSS: 7.G.A.2)
3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in cross sections of right rectangular prisms and right rectangular pyramids. (CCSS: 7.G.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Investigate what side and angle measurements are necessary to determine a unique triangle. (Entrepreneurial Skills: Inquiry/Analysis)
2. Reason abstractly by deconstructing three-dimensional shapes into two-dimensional cross-sections. (MP2)
3. Describe, analyze, and generalize about the resulting cross-section of a sliced three-dimensional figure and justify their reasoning. (MP3)
4. Appropriately use paper, pencil, ruler, compass, protractor, or technology to draw geometric shapes. (MP5)

Inquiry Questions:

1. How are proportions used to solve problems involving scale drawings?
2. What are some examples of cross-sections whose shapes may be identical but are from different three-dimensional figures?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In Grade 6, students solve real-world and mathematical problems involving area, surface area, and volume.
3. In Grade 7, this expectation connects with analyzing proportional relationships and using them to solve real-world and mathematical problems.
4. In Grade 8, students understand the connections between proportional relationships, lines, and linear equations, and understand congruence and similarity using physical models, transparencies, or geometry software. In high school, students use geometric constructions as a basis for geometric proof.

COLORADO
Department of Education

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP4. Model with mathematics.
MP6. Attend to precision.

Grade Level Expectation:

7.G.B. Geometry: Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

Evidence Outcomes

Students Can:

4. State the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. (CCSS: 7.G.B.4)
5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multistep problem to write and solve simple equations for an unknown angle in a figure. (CCSS: 7.G.B.5)
6. Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. (CCSS: 7.G.B.6)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Solve problems involving angle measure, area, surface area, and volume. (Entrepreneurial Skills: Inquiry/Analysis)
2. Persevere with complex shapes by analyzing their component parts and applying geometric properties and measures of area and volume. (MP1)
3. Model real-world situations involving area, surface area, and volume. (MP4)
4. Reason accurately with measurement units when calculating angles, circumference, area, surface area, and volume. (MP6)

Inquiry Questions:

1. How can the formula for the area of a circle be derived from the formula for the circumference of the circle?
2. What are the angle measure relationships in supplementary, complementary, vertical, and adjacent angles?
3. What are some examples of real-world situations where one would need to find (a) area, (b) volume, and (c) surface area?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In previous grades, students understand concepts of angle, measure angles, and solve real-world and mathematical problems involving area, surface area, and volume.
3. In Grade 8, students understand congruence and similarity using physical models, transparencies, or geometry software, and understand and apply the Pythagorean Theorem. Students also use the formulas for the volumes of cones, cylinders, and spheres to solve real-world and mathematical problems.