Colorado
Academic Standards

Mathematics

COLORADO
Department of Education

ALL STUDENTS • ALL STANDARDS

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Purpose of Mathematics

"Pure mathematics is, in its way, the poetry of logical ideas."

~Albert Einstein, *Obituary for Emmy Noether* (1935)

"Systematization is a great virtue of mathematics, and if possible, the student has to learn this virtue, too. But then I mean the activity of systematizing, not its result. Its result is a system, a beautiful closed system, closed with no entrance and no exit. In its highest perfection it can even be handled by a machine. But for what can be performed by machines, we need no humans. What humans have to learn is not mathematics as a closed system, but rather as an activity, the process of mathematizing reality and if possible even that of mathematizing mathematics."

~Hans Freudenthal, *Why to Teach Mathematics So as to Be Useful* (1968)

Mathematics is the human activity of reasoning with number and shape, in concert with the logical and symbolic artifacts that people develop and apply in their mathematical activity. The National Council of Teachers of Mathematics (2018) outlines three primary purposes for learning mathematics:

1. To Expand Professional Opportunity. Just as the ability to read and write was critical for workers when the early 20th century economy shifted from agriculture to manufacturing, the ability to do mathematics is critical for workers in the 21st-century as the economy has shifted from manufacturing to information technology. Workers with a robust understanding of mathematics are in demand by employers, and job growth in STEM (science, technology, engineering, and mathematics) fields is forecast to accelerate over the next decade.

2. Understand and Critique the World. A consequence of living in a technological society is the need to interpret and understand the mathematics behind our social, scientific, commercial, and political systems. Much of this mathematics appears in the way of statistics, tables, and graphs, but this need to understand and critique the world extends to the application of mathematical models, attention given to precision, bias in data collection, and the soundness of mathematical claims and arguments. Learners of mathematics should feel empowered to make sense of the world around them and to better participate as an informed member of a democratic society.

3. Experience Wonder, Joy, and Beauty. Just as human forms and movement can be beautiful in dance, or sounds can make beautiful music, the patterns, shapes, and reasoning of mathematics can also be beautiful. On a personal level, mathematical problem solving can be an authentic act of individual creativity, while on a societal level, mathematics both informs and is informed by the culture of those who use and develop it, just as art or language is used and developed.

References

National Council of Teachers of Mathematics (2018). *Catalyzing change in high school mathematics: Initiating critical conversations*. Reston, VA: National Council of Teachers of Mathematics.

Prepared Graduates in Mathematics

Prepared graduates in mathematics are described by the eight *Standards for Mathematical Practice* described in the Common Core State Standards:

MP1. Make sense of problems and persevere in solving them.

MP2. Reason abstractly and quantitatively.

MP3. Construct viable arguments and critique the reasoning of others.

MP4. Model with mathematics.

MP5. Use appropriate tools strategically.

MP6. Attend to precision.

MP7. Look for and make use of structure.

MP8. Look for and express regularity in repeated reasoning.

Standards in Mathematics

The Colorado Academic Standards in mathematics are the topical organization of the concepts and skills every Colorado student should know and be able to do throughout their preschool through twelfth grade experience. The standards of mathematics are:

1. Number and Quantity
From preschool through high school, students are continually extending their concept of numbers as they build an understanding of whole numbers, rational numbers, real numbers, and complex numbers. As they engage in real-world mathematical problems, they conceive of quantities, numbers with associated units. Students learn that numbers are governed by properties and understand these properties lead to fluency with operations.

2. Algebra and Functions
Algebraic thinking is about understanding and using numbers, and students' work in this area helps them extend the arithmetic of early grades to expressions, equations, and functions in later grades. This mathematics is applied to real-world problems as students use numbers, expressions, and equations to model the world. The mathematics of this standard is closely related to that of Number and Quantity.

3. Data Analysis, Statistics, and Probability
From the early grades, students gather, display, summarize, examine, and interpret data to discover patterns and deviations from patterns. Measurement is used to generate, represent and analyze data. Working with data and an understanding of the principles of probability lead to a formal study of statistics in middle in high school. Statistics provides tools for describing variability in data and for making informed decisions that take variability into account.

4. Geometry
Students' study of geometry allows them to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, and engage in logical reasoning. Students learn that geometry is useful in representing, modeling, and solving problems in the real world as well as in mathematics.

Modeling Across the High School Standards

A star symbol (★) in the high school standards represents grade level expectations and evidence outcomes that make up a mathematical modeling standards category.

Modeling links classroom mathematics and statistics to everyday life, work, and decision making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards.

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Prepared Graduates:

MP7. Look for and make use of structure.
MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

K.CC.A. Counting & Cardinality: Use number names and the count sequence.

Evidence Outcomes

Students Can:

1. Count to 100 by ones and by tens. (CCSS: K.CC.A.1)
2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1). (CCSS: K.CC.A.2)
3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects). (CCSS: K.CC.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Recognize that the number sequence from 1 to 9 repeats between the decade numbers, except in the spoken numbers between 10 and 20. (MP7)
2. Reason that counting to 100 by tens reaches the same number as can be counted repeatedly by ones. (MP8)

Inquiry Questions:

1. When might you want to count by tens instead of ones?
2. When might you want to start counting from a number other than one?
3. What number can we use to show we have nothing to count?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In preschool, students understand that number words have a sequence and that the words are separate (not “onetwothree”).
3. In kindergarten, this expectation is key to several progressions of learning: (a) from saying the counting words to counting out objects, (b) from counting to counting on, and (c) from spoken number words to written base-ten numerals to base-ten system understanding.
4. In Grade 1, students extend the counting sequence to 120.

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Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP3. Construct viable arguments and critique the reasoning of others.
MP7. Look for and make use of structure.

Grade Level Expectation:

K.CC.B. Counting & Cardinality: Count to determine the number of objects.

Evidence Outcomes

Students Can:

4. Apply the relationship between numbers and quantities and connect counting to cardinality. (CCSS: K.CC.B.4)
   a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. (CCSS: K.CC.B.4.a)
   b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. (CCSS: K.CC.B.4.b)
   c. Understand that each successive number name refers to a quantity that is one larger. (CCSS: K.CC.B.4.c)
5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects. (CCSS: K.CC.5)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Progress from thinking about numbers as the result of the process of counting to abstractly thinking about numbers as mental objects of their own—especially the quantity 10. (MP2)
2. Explain how the number reached when counting on is a relationship between the quantity started from and the quantity added. (MP3)
3. Make counting efficient by following rows, columns, or other patterns in a group of arranged objects. (MP7)

Inquiry Questions:

1. How is counting to five different from the number five?
2. What number is one larger than four? What number is one larger than seven?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In preschool, students build conceptions of what whole numbers mean, of subitizing, of one-to-one correspondence between verbal counting and objects, and of cardinality.
3. In kindergarten, this expectation is key to several progressions of learning: (a) from saying the counting words to counting out objects, (b) from counting to counting on, and (c) from spoken number words to written base-ten numerals to base-ten understanding.
4. In Grade 1, students use their understanding of counting and cardinality to add and subtract within 20.

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Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP6. Attend to precision.

Grade Level Expectation:

K.CC.C. Counting & Cardinality: Compare numbers.

Evidence Outcomes

Students Can:

6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. (Include groups with up to 10 objects.) (CCSS: K.CC.C.6)
7. Compare two numbers between 1 and 10 presented as written numerals. (CCSS: K.CC.C.7)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make reasoned arguments about the relative sizes of groups, such as by matching objects of two groups and seeing which has extra objects, or by counting the objects in each group and seeing which has the number further in the counting sequence. (MP3)
2. Use precise language to describe why one quantity is less than, greater than, or equal to another, and avoid mixing and misusing different ways of quantifying such as dimension, weight, or magnitude. (MP6)

Inquiry Questions:

1. Other than counting, how might you decide whether one set has more objects than another?
2. Which is more, 3 small cookies or 2 big cookies? What makes this difficult to answer?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In preschool, students build an understanding of same versus different numbers of items, numbers of objects versus their size, and ordering from first to fifth.
3. In kindergarten, this expectation is key to several progressions of learning: (a) from counting to counting on and (b) from comparison by matching to comparison by numbers to comparison involving adding and subtracting.
4. In Grade 1, students build an understanding of ten and place value with two-digit numbers. Students also organize data into categories and compare how many more or less are in one category than in another.

Prepared Graduates:

MP6. Attend to precision.
MP7. Look for and make use of structure.
MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

K.NBT.A. Number & Operations in Base Ten: Work with numbers 11–19 to gain foundations for place value.

Evidence Outcomes

Students Can:

1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as $18 = 10 + 8$); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones. (CCSS: K.NBT.A.1)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Be precise in drawings, diagrams, and numerical recordings about objects or symbols that represent ones and objects or symbols that represent tens. (MP6)
2. See the structure of a number as composed of its base-ten units. (MP7)
3. Repeat the reasoning afforded by the uniformity of the base-ten system, where 10 copies compose 1 base-ten unit of the next highest value. (MP8)

Inquiry Questions:

1. Can you show the number 13 as ten ones and some more ones? How many more ones than tens are there?
2. In the number 11, what makes the "1" on the left different from the "1" on the right? Could you show this with objects or a diagram?
3. What would a number called "ten four" look like? What word do we usually say for this number?
4. Why might someone call the number 17 "ten seven?"

Coherence Connections:

1. This expectation represents major work of the grade.
2. In preschool, students develop conceptions of addition and subtraction when adding to and taking away from small collections of objects.
3. In kindergarten, this expectation is part of a progression from comparison by spoken number words to written base-ten numerals to base-ten system understanding.
4. In Grade 1, students build an understanding of ten and place value with two-digit numbers.

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Prepared Graduates:

MP4. Model with mathematics.
MP5. Use appropriate tools strategically.
MP6. Attend to precision.

Grade Level Expectation:

K.OA.A. Operations & Algebraic Thinking: Model and describe addition as putting together and adding to, and subtraction as taking apart and taking from, using objects or drawings.

Evidence Outcomes

Students Can:

1. Represent addition and subtraction with objects, fingers, mental images, drawings (drawings need not show details, but should show the mathematics in the problem), sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. (CCSS: K.OA.A.1)
2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. (CCSS: K.OA.A.2)
3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $5 = 2 + 3$ and $5 = 4 + 1$). (CCSS: K.OA.A.3)
4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation. (CCSS: K.OA.A.4)
5. Fluently add and subtract within 5. (CCSS: K.OA.A.5)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make sense of real-world situations involving addition and subtraction (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Mathematize a real-world situation, focusing on the quantities and their relationships rather than non-mathematical aspects of the situation. (MP4)
3. Act out adding and subtracting situations by representing quantities in the situation with objects, fingers, and math drawings. (MP5)
4. Use the equal sign consistently and appropriately. (MP6)

Inquiry Questions:

1. How could you show me adding 3 and 2?
2. How could you show me 3 take away 2?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In preschool, students represent addition and subtraction within 5 with fingers, objects, and drawings.
3. In kindergarten, this expectation is part of a progression involving addition and subtraction of increasingly large numbers and increasingly complex problem subtypes (see Appendix, Table 1).
4. In Grade 1, students understand properties of operations, the relationship between addition and subtraction, and add and subtract within 20.

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Prepared Graduates:

MP6. Attend to precision.

Grade Level Expectation:

K.MD.A. Measurement & Data: Describe and compare measurable attributes.

Evidence Outcomes

Students Can:

1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object. (CCSS: K.MD.A.1)
2. Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.* (CCSS: K.MD.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make sense of their world by comparing and ordering objects by their attributes. (Entrepreneurial Skills: Inquiry/Analysis)
2. Be precise about meanings related to size when describing an object's height, weight, or other attribute. (MP6)

Inquiry Questions:

1. What does it mean for one object to be "bigger" than another?
2. If you are standing on a chair, how should your height be measured differently than if you were standing on the floor?
3. If an object is moved, does that change its size?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In preschool, students develop conceptions of measurable attributes of objects and comparisons based on those attributes.
3. In kindergarten, this expectation can contribute to students' understandings of measurable attributes, comparison, and conservation of length, all of which connect to progressions in geometry, the number system, and to future work in ratio and proportion.
4. In Grade 1, students measure lengths directly and by iterating length units, and express the length of an object as a whole number of length units.

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Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP5. Use appropriate tools strategically.

Grade Level Expectation:

K.MD.B. Measurement & Data: Classify objects and count the number of objects in each category.

Evidence Outcomes

Students Can:

3. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. (Limit category counts to be less than or equal to 10.) (CCSS: K.MD.B.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Group objects into categories to help make sense of problems. (MP1)
2. Abstract individual objects into new conceptual groups. (MP2)
3. Choose appropriate representations of objects and categories. (MP5)

Inquiry Questions:

1. How can numbers of objects be represented to make comparisons?
2. How can objects be categorized in different ways?
3. How can an object's attributes determine if it does not belong with other objects in a group?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In preschool, students use differences in attributes to make comparisons.
3. In kindergarten, this expectation supports the work of counting and comparing numbers and is part of a progression of learning how to analyze categorical data.
4. In Grade 1, students organize, represent, and interpret data with up to three categories.

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Prepared Graduates:

MP4. Model with mathematics.
MP6. Attend to precision.
MP7. Look for and make use of structure.

Grade Level Expectation:

K.G.A. Geometry: Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

Evidence Outcomes

Students Can:

1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*. (CCSS: K.G.A.1)
2. Correctly name shapes regardless of their orientations or overall size. (CCSS: K.G.A.2)
3. Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid"). (CCSS: K.G.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Describe the physical world from geometric perspectives, e.g., shape, orientation, and spatial relationships. (MP4)
2. Reflect an increasing understanding of shapes by using increasingly precise language to describe them. (MP6)
3. Sort shapes into categories (squares, circles, triangles, etc.) based on attributes of the shapes. (MP7)

Inquiry Questions:

1. For a given shape, what attributes make an example of that shape different from a non-example? For example, "Why is this shape (point to a square) a square, while this shape (point to a non-square) is not?"
2. What are the ways of describing where an object is?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In preschool, students learn about circles, squares, triangles, and their parts.
3. In kindergarten, this expectation connects with the work of analyzing, comparing, creating, and composing shapes.
4. In future grades, students calculate area and surface area of these and other shapes.

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Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.
MP7. Look for and make use of structure.

Grade Level Expectation:

K.G.B. Geometry: Analyze, compare, create, and compose shapes.

Evidence Outcomes

Students Can:

4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length). (CCSS: K.G.B.4)
5. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes. (CCSS: K.G.B.5)
6. Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"* (CCSS: K.G.B.6)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Use experiences with multiple examples of a type of shape to develop a concept image (see glossary) of that shape from which they can abstract common features. (MP2)
2. Model shapes in the world by building them with components or drawing representations of them. (MP4)
3. Use patterns or structures when making comparisons or compositions of shapes. (MP7)

Inquiry Questions:

1. Can you change a shape into a different kind of shape by rotating it?
2. What kinds of pictures can you make by combining shapes?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In preschool, students understand and use language related to directionality, order, and the position of objects, such as up/down and in front/behind.
3. In kindergarten, this expectation connects with identifying and describing shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).
4. In Grade 1, students classify, compose, and partition shapes.